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MAPS

THEIR USES AND CONSTRUCTION

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PREFACE

TO

THE SECOND EDITION

To those critics of the first edition who have reviewed it favourably I have to return my sincere thanks.

The adverse criticisms, with one exception, may be condensed as follows:

That it is to be regretted that the work is not more complete; that the method of Stereographic construction might be more clearly expressed; that the explanation of the principle of Mercator's projection is disappointing; that the description of Conical projection is far from being exhaustive, and that Elliptical projection might have been referred to; also that tables of meridional parts and of lengths of degrees might have been added.

The justice of these criticisms I fully admit, but it is satisfactory to see that the faults are blemishes which may be removed, rather than errors which need correction.

The book was written at the suggestion of an officer of high standing in charge of survey work, who,

in the course of conversation on a long sea voyage, expressed his opinion very strongly, to the effect that an easy and elementary book on map projection would be of great use to many junior members of his survey staff, who found it very difficult to understand the regular treatises, and who often got lost in a mass of details.

In response to this suggestion the book was written at the other side of the globe, with scarcely anything to refer to, with no one to check calculations, with the necessity of constructing many of the diagrams with my own hands, and with the disadvantage of having the book printed without even seeing a proof; and while my publisher has done all that could be expected, or more than could have been expected, in the way of accuracy and general get up, the fact still remains that matter in type sometimes looks different from matter in manuscript.

Having now the advantage of writing in London, where I have been enabled to consult previous works, and where I have had the good fortune of being able to discuss the subject with Mr. Reeves, of the Royal Geographical Society, from whom I have received many valuable suggestions, I have done my best to correct the shortcomings of the first edition.

The first chapter I have scarcely touched; the second chapter I have revised, and I have added a description of Elliptical projection; the third chapter I have revised and rearranged, and I have added a considerable amount of new matter, besides improving the descriptions of Stereographic and Mercator's projections, and to some extent expanding the description of Conical projection; the fourth chapter I have entirely

rewritten. As regards plates I have discarded five of the original figures and added seventeen new ones.

I have also added some tables of lengths of degrees, &c., as suggested by my critics, but I have done this with serious misgivings, as I may be opening the door to having the work compared with others, to an equality with which it lays no claim. The only treatise with which I am acquainted which is on similar lines is that of Professor Hughes, which treatise I had never seen when I wrote the first edition.

In carrying out the work I have tried to make everything but accuracy subservient to simplicity, and with this end in view I have constantly treated specific examples as if they were general cases. From a mathematical standpoint this is perhaps indefensible, but it is better that a young student should clearly understand a specific example than that he should get utterly fogged over a general case, and with beginners the choice often lies between these two. I have moreover brushed aside an immense amount of detail. Such detail is of two sorts. First, there is the mathematical detail of such things as errors of shape and size in various projections. These require a considerable amount of mathematical knowledge, and I firmly believe look much more important when stated in the form of figures than when dealt with in actual map making; and, as the book professes to be an introduction to the latter art and not to the former science, I make no apologies for passing them over. In the second place there is the practical detail. Suppose the latitude and longitude of one point be known, and also the azimuth and distance of another

point: a long and laborious calculation, requiring the consideration of some thirty or forty different quantities, is necessary to determine the latitude and longitude of the latter point with what may be called absolute accuracy. Unless this is attended to, all triangulation is labour thrown away and attention to the most minute details is imperative; but I have omitted all reference to such details as these in spite of their importance, simply because they are clearly outside the limits of the treatise.

The treatise is addressed as regards the first and second chapters to the general public, and as regards the third and fourth chapters to those requiring a superficial knowledge of the subject, as well as to those who require an introduction to its serious treatment. It naturally must not be compared with such works as those of Germain or of Colonel Clarke in the 'Encyclopædia Britannica,' which stand altogether on a much higher plane.

The criticism which I have not included in my condensed statement is to the effect that some of my opinions may not meet with universal approval. This is very likely true. I have tried, and I hope successfully, to keep my opinions clear of my statements of fact, but I see no reason to modify them. Such as they are, they are my own, and must be taken for what they are worth.

G. JAMES MORRISON.

London, 1902.

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MAPS

THEIR USES AND CONSTRUCTION

CHAPTER I

INTRODUCTORY

EVERYONE from time to time makes use of maps, but hardly anyone ever consults a globe, and it is not going too far to say that in the case of ninety-nine people out of a hundred all their ideas of the relative sizes, shapes, and positions of the various countries in the world are derived entirely from maps, and generally from those on Mercator's projection; yet of these people probably not one in ten knows that the ideas so derived are inaccurate to a degree which may fairly be called misleading.

On Mercator's maps the size of all polar countries is enormously exaggerated, Greenland, for instance, appearing to be many times the size of India, whereas it has approximately the same area; while as regards position, the direct route from London to Shanghai appears to pass through the Caspian Sea, though as a matter of fact it passes north of St. Petersburg. In these and various other ways maps not only on Mercator's

projection, but on any other projection, are and must be faulty.

It is proposed in the following pages, in the first place to explain in a popular way why it is that no absolutely correct map can be made, and what are the good and bad points of maps constructed on various systems ; and to follow this up with a general explanation of the mathematical bases of the most useful projections, for the benefit of those interested in this branch of the subject.

It is hardly necessary to state that no difficulties of any magnitude are encountered in making maps of moderate areas. A flat map of England may be made, which, for the purpose of studying geography, may be looked upon as absolutely accurate ; but a map showing half the world or the whole world must be distorted to a considerable extent. While, however, this is true, it is also true that in such maps it is generally possible to preserve some one important quality by sacrificing others. Thus (1) a map may be made on which all small portions of the world retain very nearly their proper shapes, but countries near the edges of the map are on a larger scale than those near the centre. Or (2) a map may be made on which all countries retain their areas accurately, but the shape of each is somewhat distorted. Again (3) a map may be made on which all points retain the proper compass bearings of the routes between them ; that is to say, if on the map one port is north-east of another port, then a ship starting from the latter and sailing north-east will reach the former, but the course will not be the shortest possible : a state-

ment which will be made clear hereafter. Or (4) a map may be made on which a straight line drawn from one point to another will indicate accurately the shortest route on the surface of the globe ; but in both these maps, (3) and (4), the distortion near the edges is very great.

The first-named map is very commonly used for giving people general ideas ; that is to say, for teaching geography. The second class, of which there are many varieties, is useful for giving good ideas as to relative size, and as the distortion of shape in some of these is not very great they deserve to be in general use, while, as a matter of fact, they are hardly known. The third, or Mercator's map, is useful as a sailing chart for moderate voyages, and has the advantage of showing the entire inhabited world on one sheet, a quality which has led to its being used for general purposes to an extent which is quite unjustified. The fourth is useful for laying down long ocean routes, but as it is not on the whole a convenient sailing map it is much less used than it ought to be.

It may seem strange to begin a treatise on maps by what may appear to be a general condemnation of the entire species. The fact is that maps of the world, or of large portions of the world, are at best miserable make-shifts ; but as they are absolutely necessary for many purposes, it has been thought it would be useful if the principles on which they are founded could be explained in a manner that would be clear to those who have not made a study of mathematics, so that they might see for themselves what are the good and bad points of various projections, and so be on their guard against

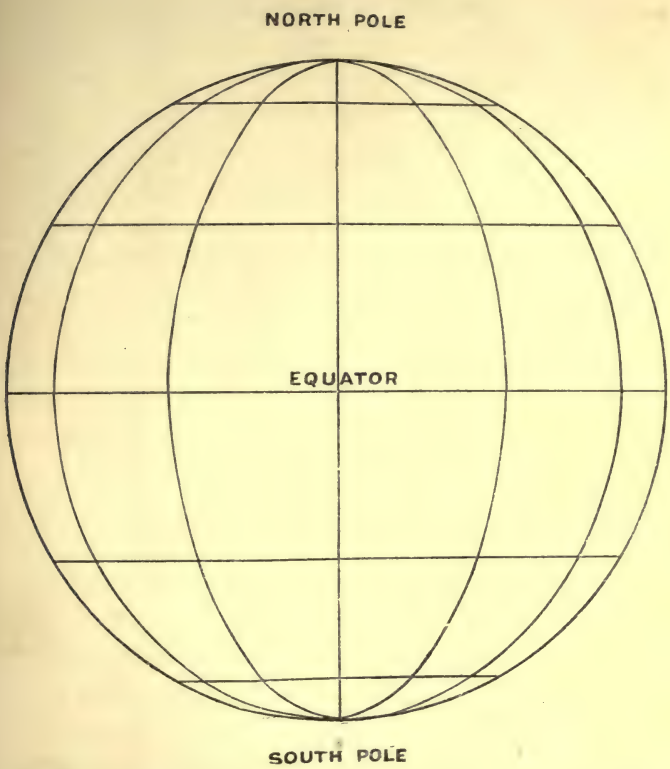
using a map for a purpose for which it is not suited, and so being misled, and badly misled, by appearances.

As everyone knows, the earth is a sphere, and though it is slightly flattened at the poles it will be assumed in this portion of the treatise, as is usual in general treatises on projections, that it is a perfect sphere. An accurate model of the earth may therefore be made in the form of a globe, but the inconvenience of using globes is so great that people are practically driven to flat maps. Now, if a correct representation of the world can be made on a globe, the simplest way in principle to make a flat map would be to lay a piece of tracing paper on the globe and trace the map through. A single attempt will show the futility of this method. Unless a very small piece of paper be used, it will not lie flat. Of course no piece, however small, will lie absolutely flat, but a comparatively small piece will lie so close to the surface of the globe that no appreciable crumpling takes place, and the portion of the globe covered can be traced so as to give a flat map without appreciable error. The larger the globe the larger the piece of paper that can be used, but the larger also will be the countries, so that the area to be thus shown cannot be increased. A map of the British Isles might perhaps with difficulty be accomplished, but if one tries to cover half the globe one finds the paper crumple up so as to make tracing impossible. Now this difficulty lies at the root of all map-making, and it cannot be abolished; but there are certain methods by which, to a limited extent, it can be evaded.

The positions of places on the earth are identified by

latitude and longitude. Practically all educated people know what is meant by these words, and some readers may feel inclined to skip the next few pages, but it is advisable to explain how the latitude and longitude of

FIG. 1



places are determined in order to make the succeeding pages of this treatise more easily understood. The surface of a sphere begins and ends nowhere. Any point on it bears the same relation to the whole that

every other point bears, and there is no reason to choose one point more than another to start from. In the case of the earth, however (see fig. 1), there is something. It revolves on its axis once a day, and this axis is therefore a definite line, the ends of which are called poles, one being called North and the other South. Here, then, is something definite to go upon. The earth is supposed to be divided into two equal parts by a line which goes round the world, half-way between the poles, and is called the Equator. The position of this, being defined with reference to the poles, is likewise definite. All circles which divide a sphere into two equal parts are called great circles, and the Equator is therefore a great circle. It has for a long time been the habit to divide a circle into 360 equal parts, called degrees. There is no reason why 360 should have been chosen rather than 400 or 100 or any other number, but it has been used so long that one has come to look upon it as a natural division. 90 degrees therefore means one fourth of a circle, 45 degrees means one eighth, 60 degrees means one sixth, and so on, a degree being indicated by a small circle after the figure, thus: 60°.

Having decided to divide the Equator into 360 degrees, one is met by the difficulty of having no point at which to begin. It will be better to leave this question for a moment and return to it.

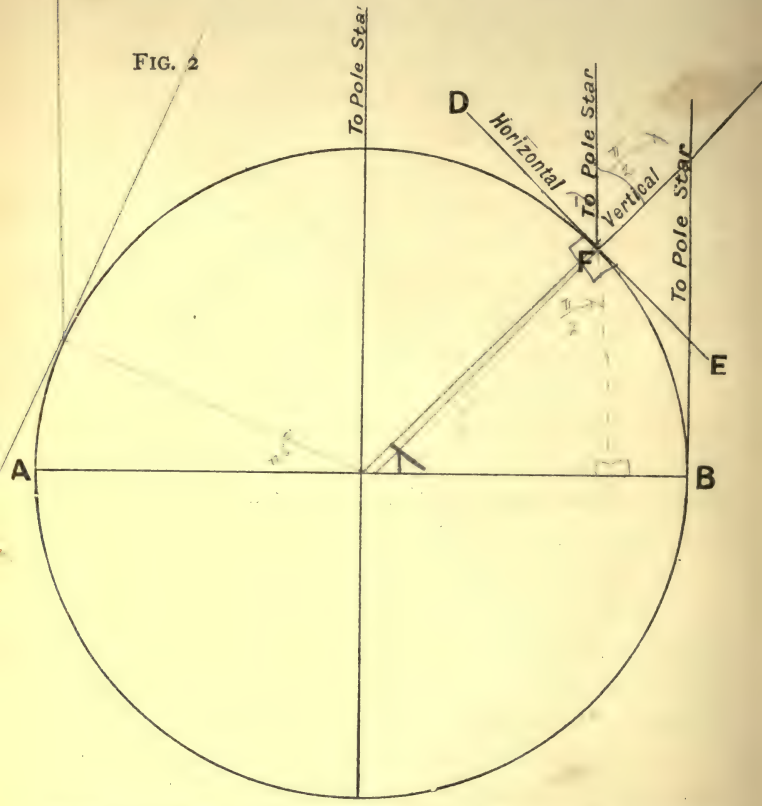
It is clear that any number of great circles can be drawn through the two poles, each of which will cut the Equator into two equal parts. Each of these may be divided into 360°, and there will be 90° between the Equator and each pole on each side. It is usual to

number these degrees from the Equator towards the pole, the Equator being 0° and the pole 90°. Now suppose some point is chosen measuring 10° north of the Equator on one of these great circles (which, as will be seen later on, are called meridians) and a circle is drawn through it parallel to the Equator, this circle will be the parallel of 10° north latitude and will cut every meridian 10° north of the Equator. In the same way other circles may be drawn at 20°, 30°, &c., to represent parallels of 20° and 30° of latitude, and further circles may be drawn at every 10° or at every degree north and south, and all of these will be parallels of latitude. It is clear that each of these parallels of latitude cuts the sphere, not into two equal parts, but into two unequal parts. The parallels of latitude are therefore not great circles, but small circles. The point to be established, however, is that the Equator has been drawn with reference to the poles and the parallels have been drawn with reference to the Equator, and are therefore definite in position. There is absolutely nothing arbitrary except the numbering of the degrees. The parallel which lies half-way between the Equator and the pole is called the parallel of 45°. It might be called by another name, but none the less it is a definite line on the surface of the globe, and not an arbitrary line.

The latitude of any place is determined on the following principles. There happens to be a star very nearly in the prolongation of the earth's axis to the north, to which the name pole-star is given. Suppose anyone to be at the North Pole, this star would appear to him to be directly overhead. Suppose, again, a person

to be at the Equator, the star would appear to him to be on the horizon, level with his eye. It might be said that it would appear below his eye, because it is in line with the axis of the earth, 4,000 miles below his feet, but the

FIG. 2



distance of the star is so enormous that this is nothing. All lines to it from any point on the earth appear to be parallel. Suppose a person to be at F (see fig. 2), halfway between the Equator and the pole, the inclined

line on the earth, D E, will appear to him to be horizontal, and he will see the star half-way up the heavens. Now this is equivalent to saying that the latitude of any place is indicated by the height of the pole-star. Most people who have travelled much have observed as they go south that the pole-star night by night appears lower in the heavens and gradually disappears while the Southern Cross gradually comes into view. It is true that at sea the latitude is fixed every day at noon by an observation of the sun, but this is because the sun is brighter and more easily observed. Its distance from the pole, which varies throughout the year, is tabulated for each day in a book called the Nautical Almanac, and when it is observed its polar distance is allowed for, and what is actually stated as the latitude of the ship is in fact the height of the pole in the heavens, and the star itself is as a matter of fact directly observed from time to time. This shows that latitude is not arbitrary. If the star appears one fourth of the way up, measured from the horizon towards the zenith, then the point of observation is one fourth of the way up from the Equator towards the pole, and nothing can alter this. It is of course to be understood that when the pole-star is spoken of, the pole (the prolongation of the earth's axis), to which the star is close, is really meant. When the star itself is observed the difference is allowed for. Southern latitudes are based on the same principles.

Strictly speaking, this is what is called the astronomical latitude of a place. There are, as will be shown later, other latitudes which slightly differ from the above, partly because the earth is not a true sphere and

partly from local attractions, but the above described latitude is not only the one adopted in all general treatises, but it is the one adopted on all general maps and charts, whether issued by the Admiralty or by others, and is the latitude by which all navigation is conducted; and, assuming the earth to be a perfectly uniform sphere, it is the only latitude.

This, however, fixes only the parallel of latitude on which the point of observation is situated. If it be found that the latitude of one place is 10° north and the latitude of another 20° north, then the second place is 10° north of the former, but there is nothing to show whether it is east or west of it.

Suppose at any point on the earth's surface a perpendicular pole be erected. The shadow of the pole in the morning will be on the west side, and in the evening on the east side, and there is a certain moment when it will lie due north and south. That moment is called noon, and it will be the same for all points exactly north or south of the point of observation. A great circle passing through the poles of the earth and through the point of observation is called a meridian (from *meridies*, mid-day), and it is therefore noon at the same moment at all points on that meridian. Suppose a chronometer keeping correct time to be set to noon and then carried to some other point of observation and noon observed at the new point and compared with the chronometer, and suppose it is found that at noon of the new point the chronometer marks two o'clock, then the meridian on which the new point is situated is one-twelfth of the way round the world

to the westward of the first point. This difference is absolute and definite, there is nothing arbitrary about it, but it would be most inconvenient to have to work simply with differences of various places, and all would be chaos unless some one place was agreed upon as the starting-point. This question cropped up as soon as longitudes began to be thought of and long before they were accurately fixed, and a great many places in turn were used ; but as soon as English people began to make charts they adopted the meridian that passed through their principal observatory of Greenwich as the standard from which all longitudes should be measured, and this meridian has now been adopted by many other countries ; France, however, takes the meridian of Paris. The adoption of one meridian as standard rather than another is purely arbitrary. The divisions of the Equator are made to begin where the standard meridian crosses it, and degrees are counted 180° each way. The great circle, therefore, which passes through Greenwich is called the meridian of Greenwich or meridian 0° on one side of the globe, and the 180th meridian on the other side, it being 180° east and also 180° west of the zero meridian. By setting a chronometer to Greenwich time and observing the hour of noon at various places their longitude can be determined, allowing 15° to each hour of time, because the earth turns once on its axis in 24 hours and there are 360° in the entire circumference. It is of course to be understood that this is only a rough outline of the principles on which longitude is determined. The exact determination of longitude is a work of considerable difficulty, and the

longitudes of the principal observatories have not even at the present day been fixed with that degree of accuracy which is necessary for certain delicate observations.

It is clear that if a globe have the lines of latitude and longitude drawn on it on the principles above stated, and the latitude and longitude of a certain number of places be determined by observation, these places can be plotted in their proper positions on the globe and the detail can be filled in by ordinary surveying. If, in the same way, lines to represent latitude and longitude be drawn on a piece of paper, the places can be plotted with reference to these lines and the detail filled in by surveying as before; and the art of making maps consists in the first place in laying down the lines to represent latitude and longitude, either as nearly like the lines on the globe as is possible in transferring lines from a curved to a flat surface, or else in such a way that some one property of the lines will be retained at the expense of others. To transfer the irregular coast-lines from a globe to a map would be a work of superhuman difficulty; but to transfer regular lines is comparatively easy, and it is of course possible to lay down on a map lines corresponding to those which would be drawn on an imaginary globe many feet or even many miles in diameter. The lines of latitude and longitude may be laid down for every 10°, or every degree, or every quarter degree, or less if necessary; but the principle is—lay down the lines, plot the principal points, and fill in by surveying. After one map has been made it may of course be copied even on to another projection, care being taken that the latitude and longitude

of every point are kept correct throughout. It is clear that if the lines of latitude and longitude cannot be accurately laid down on a flat surface, still less can the detail be laid down. This difficulty is got over as follows: For the first maps such a moderate area is taken that there is no practical difference between the flat surface and the curved surface, and it will be assumed that the maps so made are bounded by lines of latitude and longitude.[1] When a large number of these maps have been made it will be found that they cannot be joined together to lie flat.

If they are carefully joined along the edges it will be found that they naturally adapt themselves to the shape of a globe; therefore another piece of paper is taken and on it are laid down lines of latitude and longitude, and each map is *copied* so as to fill the place prepared for it.

Sometimes this can be done by simple reduction which does not affect accuracy, as the accuracy of a map is independent of the scale, but sometimes the correct map has to be enlarged in one direction and reduced in another, and sometimes it has to be twisted, and in almost every case it has to be manipulated to get it into its place. The work of making maps of large areas, therefore, consists of two parts—viz., on the one hand, making correct maps of small areas, which may be called surveying; and, on the other hand, laying down lines of latitude and longitude into which these maps must be fitted, and this latter part is called map projection.

[1] The arrangement by which maps may be made of rectangular shape to join up flat over a limited area such as an English county will be dealt with in the second part of this treatise.

CHAPTER II

MAP PROJECTION CONSIDERED POPULARLY

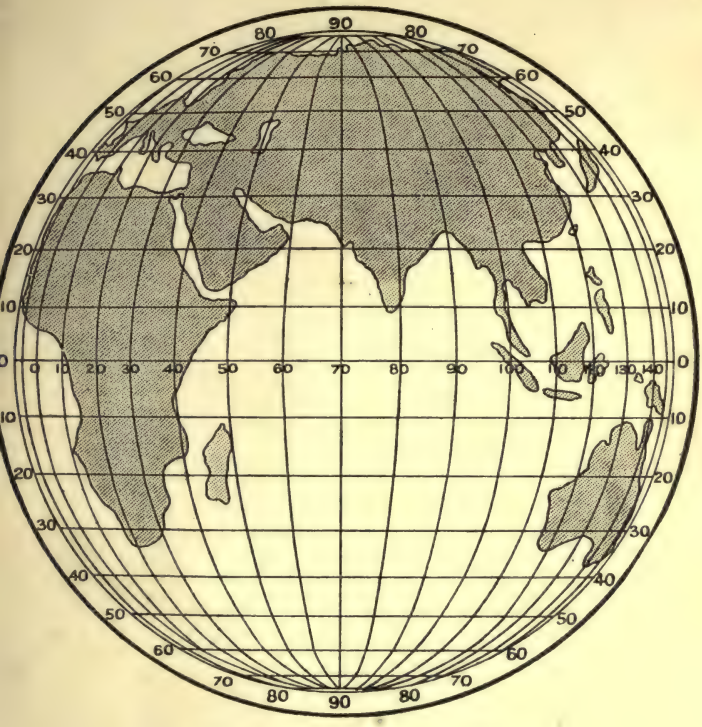
THE art of map projection, as stated above, consists in laying down lines of latitude and longitude on a flat piece of paper, either in such a way that correct maps of the various spaces can be filled in with as little distortion as possible, or else so that some one good property may be retained at the expense of others which are sacrificed. The map projector's work is therefore finished when these lines are laid down, but in the diagrams in this part of the treatise an outline of the continents will generally be added, so as to make the diagrams clearer to the ordinary reader.

The first system of projection which naturally suggests itself is to make a picture of the globe. When one looks at a globe one sees a little less than half, but if the globe is at a great distance one may assume without appreciable error that one sees half. Figs. 3 and 4 show two views of the earth on this projection, which is called ORTHOGRAPHIC. It will be seen in each case that while the centre of the map is good the edges are very much crowded, and the result is so unsatisfactory that one at once asks if it is not possible to compress the central portion into a smaller area, and so leave a little

more space for the outer portions. The projection which is now to be explained meets this difficulty to some extent.

The principle on which it is founded is shown in

FIG. 3



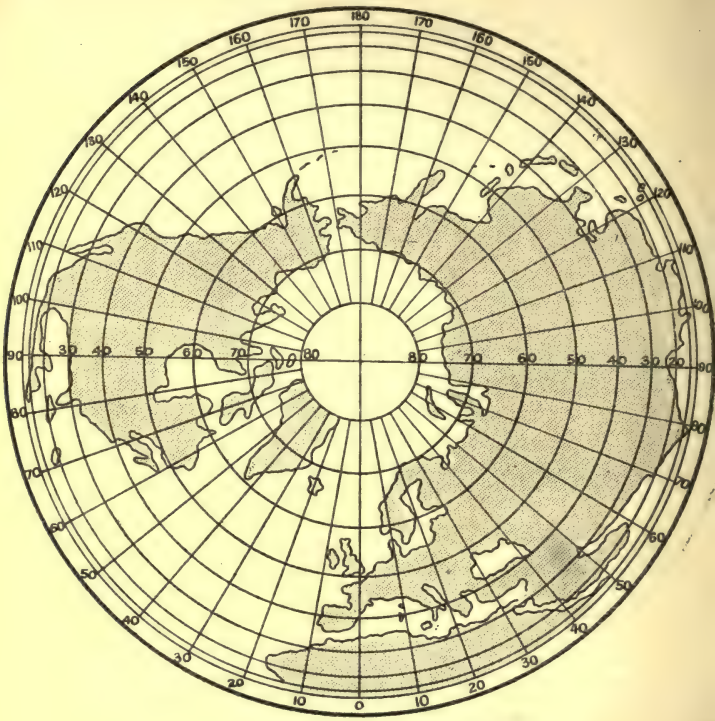
ORTHOGRAPHIC PROJECTION

fig. 5. Suppose a globe to be cut in two at A B, and a piece of paper placed between the two halves. Then suppose straight rods like knitting-needles to be run through from any points, F and H, to the point E, they will

pass through the paper at the points G and K. By using a sufficient number of points the lines of latitude and longitude can be marked out on the paper.

Three specimens of this projection, which is called STEREOGRAPHIC, are shown in figs. 6, 7, 8. The first of

FIG. 4

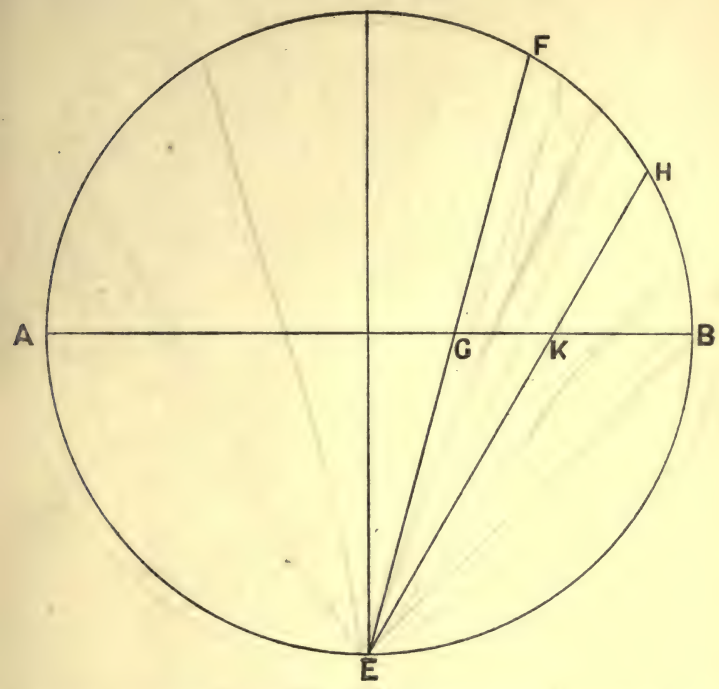


ORTHOGRAPHIC PROJECTION

these is the Northern Hemisphere, the second what is usually called the Eastern Hemisphere, and the third the hemisphere the central point of which is the intersection of the parallel of 50 with the meridian of

Greenwich (a point, that is to say, close to London). On the diagrams lines are drawn for each 10° or 20° of latitude and longitude, but it is clear that on a larger map lines could be drawn for every degree. Now, a practically correct map can be made of a portion of the

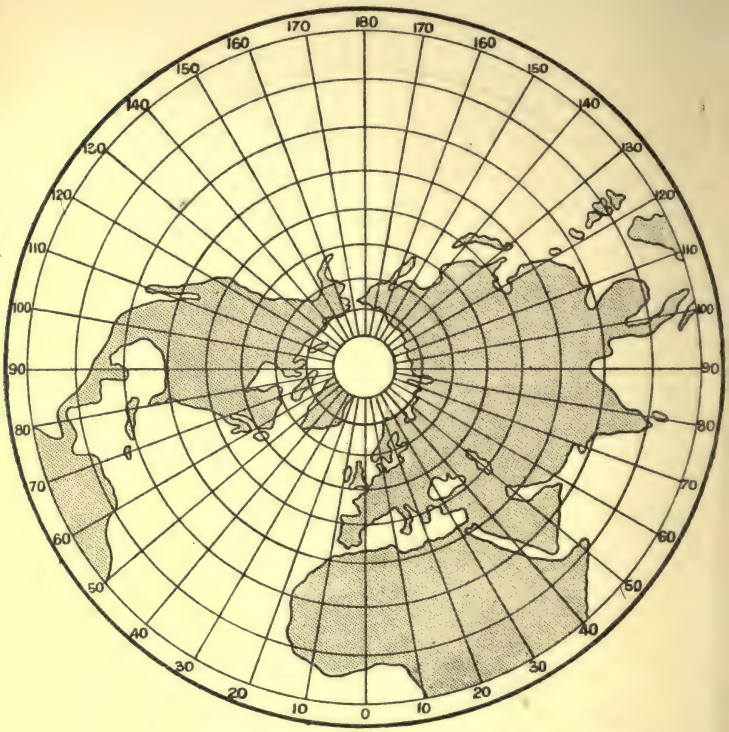
FIG. 5



world measuring 1° each way, because curvature in such a size is of no consequence. Suppose, then, that correct maps were made separately of all the little four-sided portions, it would be found that by simply reducing each of them to the requisite scale it could be fitted

almost exactly into the space to which it belonged. The words 'almost exactly' are used because the edge nearest the centre of the map would have to be com-

FIG. 6



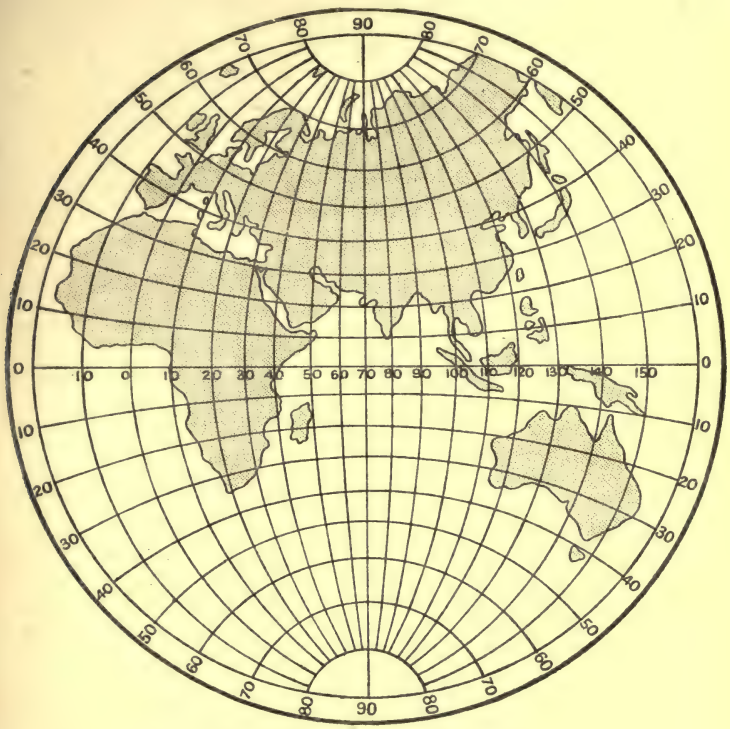
STEREOGRAPHIC PROJECTION

pressed a little, but so little that only very careful measurement would detect the difference.

Here then, at first sight, it appears that a perfect projection has been obtained, and as a matter of fact it

is considered by most authorities to be the best projection of a hemisphere for general purposes, but it has one serious defect. It has been said that each plan has to be compressed at its inner edge, and for the same

FIG. 7



STEREOGRAPHIC PROJECTION

reason each plan in succession has to be reduced to a smaller average scale than the one outside of it. In other words, the *shape* of each space into which a

plan has to be fitted is practically correct, but the *size* is less in proportion at the centre than at the edges, so that if a correct plan of an area at the edge of the map has to be reduced to a scale of 500 miles to an inch to

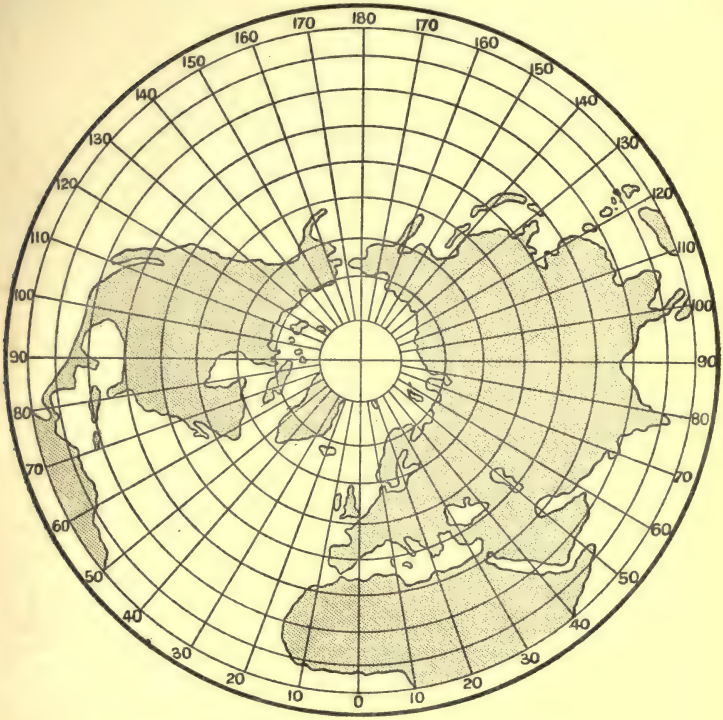
FIG. 8

STEREOGRAPHIC PROJECTION

fit its allotted space, then a plan of an area at the centre has to be reduced to a scale of 1000 miles to an inch. Thus a moderate area has its true shape, and even an

area as large as India is not distorted to an extent to be visible to the ordinary observer, but to obtain this advantage relative size is sacrificed. It is somewhat as if a ruler was represented by a billiard cue. Any short

FIG. 9



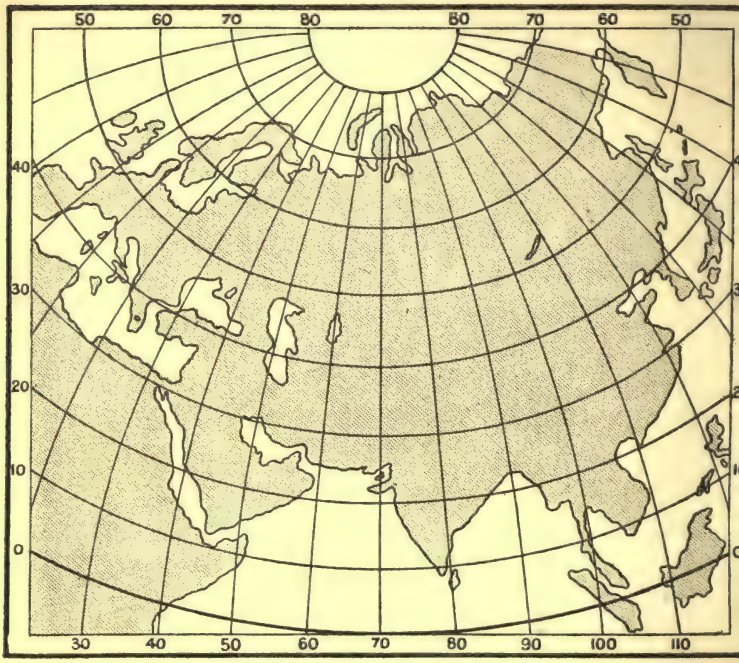
PROJECTION OF EQUAL AREAS

length of one would look like a short length of the other, but in a long length there is great dissimilarity.

The projection which next naturally presents itself

for consideration is a PROJECTION OF EQUAL AREAS. The property of equality of areas is possessed by several projections of which the following is a convenient example. In the projection just above described, area

FIG. 9A

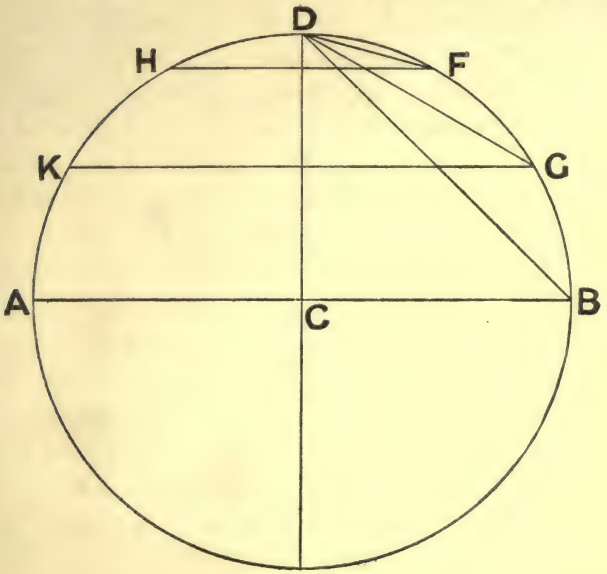


PROJECTION OF EQUAL AREAS

is sacrificed to shape. In the projection shown in figs. 9 and 9A shape is sacrificed to area, but there is this difference, that while the shape in stereographic projection is only approximately correct, the area in this projection is mathematically exact. Everyone knows

that a piece of paper measuring 3 in. by 3 in. has the same area as a piece measuring 4 in. by $2\frac{1}{4}$ in., and it is by altering shapes somewhat in this manner that true areas are obtained. The amount of distortion, though not very serious, is still sensible, as will be seen by comparing India on fig. 9 with India on Mercator's

FIG. 10

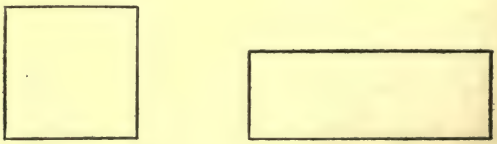


projection, where the shape of India is practically exact. But, before condemning this projection, it would be well to compare the relative sizes of India and Greenland on this map with their relative sizes on the Mercator (fig. 13). It may well be thought that the stupendous inaccuracy of Mercator as regards size is more mis-

leading than the inaccuracy of this projection as regards shape. In any case this map is useful, and should be more widely used, in order to give people correct ideas as to relative sizes, particularly seeing that, as has been stated above, the ideas of areas so obtained will be absolutely and not only approximately correct.

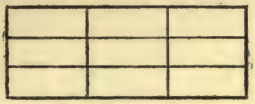
The method of constructing this projection is shown on fig. 10. Suppose on this figure D is the North Pole and A B the Equator, and H F and K G the parallels of latitude of 60° and 30° north respectively. With a pair of compasses take the length D B as radius and describe a circle. This will be the boundary of the map and will represent the Equator. With radii D F, D G, describe other circles. These will be the parallels of 60° and 30°. Describe as many more parallels in the same way as may be necessary. Draw the meridians radiating from the centre and the projection is made.

It will of course be seen that in filling in the details a difficulty arises. Suppose lines be drawn for every degree and the correct plans of the spaces be used which were used for stereographic projection. It will no longer be possible to make them fit by simply reducing the scale. What has to be done is to fit a square into a rectangle thus:



It is clear that no alteration of scale will effect this

object. What has to be done is to divide the spaces into squares and rectangles thus :



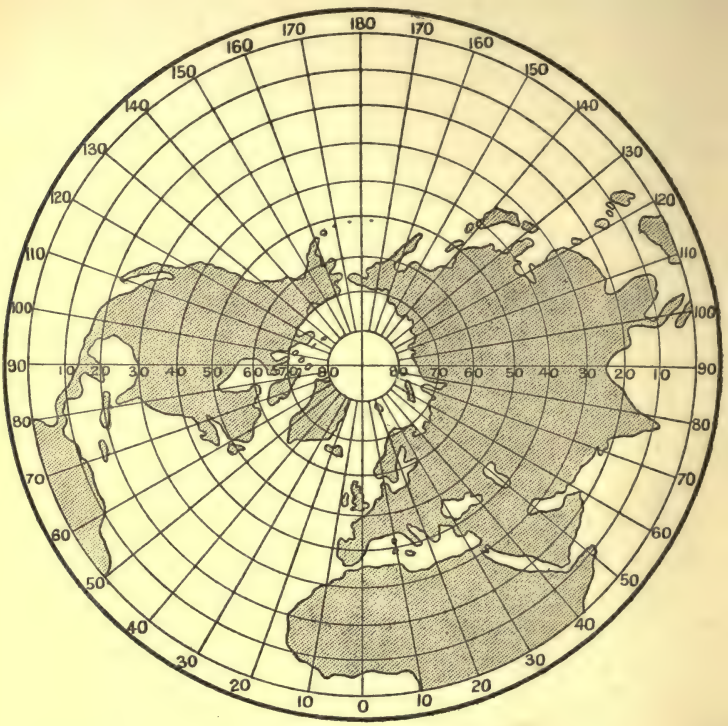
These divisions have to be made so small that a draughtsman can by eye copy into each rectangle the work shown in the corresponding square. The extent to which this dividing up is carried depends solely on the scale and the purpose for which the map is to be used. It is not suggested that it is carried to a very fine point in filling in the details in these diagrams, but the principle is the one universally adopted in copying from one projection to another.

Anyone who has followed this explanation so far will have noticed that on the stereographic polar projection the circles showing parallels of latitude are farther apart at the edge of the map than in the middle, while in the equal area projection the reverse is the case. This is not so easily seen on other hemispheres, but the principle there is the same, and the non-mathematical reader must accept the statement that the principles hold good throughout.

In the stereographic projection relative areas would be improved if the outer divisions were reduced in size and the inner divisions enlarged, and in the equal area projection shape would be improved if the outer divisions were enlarged and the inner ones reduced, and both these results would be obtained if the divisions between

parallels of latitude were made equal throughout. It is possible to construct a projection on mathematical principles in which this will be nearly the case, but it seems much simpler to adopt equal divisions at once and

FIG. 11



ARBITRARY PROJECTION

call it an ARBITRARY or CONVENTIONAL PROJECTION. This projection is shown on figs. 11 and 12. A comparison between this and the stereographic will show that shapes are not greatly distorted, while a comparison

with the equal area projection will show that areas have not been unduly sacrificed, and it seems difficult to decide definitely against this projection. It is very easily constructed, and if children were taught the geography

FIG. 12

ARBITRARY PROJECTION

of the world from projections of this sort their general ideas would probably be more correct than at present.

The hemispheres in the large atlas published by the Society for the Diffusion of Useful Knowledge appear to

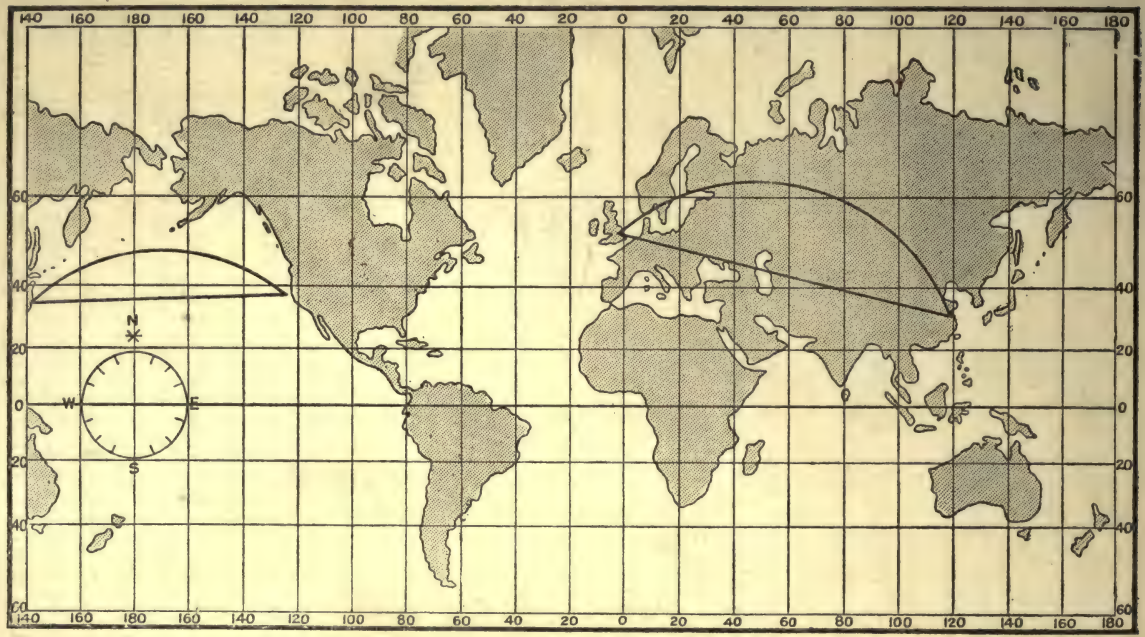
be on this projection, while those in the 'Times' Atlas are on the more generally approved stereographic projection. A comparison of the two, in the opinion of the author, inclines one to decide in favour of the arbitrary projection over the stereographic for ordinary use.

All projections of the sort hitherto considered suffer from the objection which is fatal to accuracy—viz., the diameter of the circle represents a line on the surface of the earth of exactly the same length as the line represented by the half circumference.

The projection next to be considered is MERCATOR'S (see fig. 13), and here a little preliminary explanation is necessary. It has been made quite clear that a degree means the 360th part of a circle. A degree of a small circle is therefore smaller than a degree of a large circle. All parallels of latitude are smaller than the Equator, and the parallel of 60° is exactly half its size; a degree, therefore, on that parallel is only half as long as a degree of the Equator. A portion of the earth at the Equator measuring 1° of latitude and 1° of longitude is a square each side of which measures 60 geographical miles (a geographical mile being one 60th of a degree on the meridian). A portion of the earth which measures 1° each way at latitude 60° is 60 geographical miles long measured from north to south and 30 miles measured from east to west. In any map, therefore, showing a piece of land at this latitude, if the shape is to be preserved, degrees of latitude must measure twice as much as degrees of longitude, as may be seen by looking at a map of Norway and Sweden in any atlas.

On any map, therefore, which shows a large area and

FIG. 13



MERCATOR'S PROJECTION

on which countries at the Equator have to be shown in their proper shape and also countries at 60° latitude, either the degrees of latitude must measure the same throughout and the degrees of longitude must be proportionately reduced towards the pole, or the degrees of longitude must be kept uniform and the degrees of latitude increased in proper proportion towards the pole, or the result may be effected by somewhat altering the length of both degrees, still retaining the same proportion as they have on the earth. The third method is the one adopted in stereographic projection, the second is the one adopted in Mercator's projection (which projection, however, can never be made to reach the pole), and the first method is impossible on a flat surface. On Mercator's projection a line is drawn to represent the Equator and is divided into 360 equal parts. Lines are drawn square to it at every division or every tenth division, as may be thought proper. These lines represent meridians of longitude. They are parallel to each other, and consequently degrees of longitude are of the same length all over the map. The degrees of latitude are therefore increased towards the pole so that throughout they bear to the degrees of longitude the same proportion as they do on the earth. The way in which this affects the map is well seen on fig. 13. It must be remembered that the actual distance from the Equator to the parallel of 20° is the same as the actual distance from the parallel of 60° to the parallel of 80°, but on the map the latter length is much the greater of the two. The length of a degree of latitude at latitude 60° on the map is twice as great as the length of a degree

at the Equator, while at 80° one degree of latitude is six times as large as at the Equator, and consequently if there be two pieces of land of the same area on the earth, one of them at the Equator and the other at 80° latitude, the latter on the Mercator's map is shown as having thirty-six times the area of the equatorial one. A piece of land at 89° would be shown more than 3,000 times as large as an equal-sized piece at the Equator, while a piece at the pole would be of infinite size; that is to say, the projection of the pole becomes impossible.

This excessive exaggeration of area is a most serious matter if the map be used for general purposes, and great prominence is given to this fact in this popular treatise because it is undoubtedly true that in the majority of cases people's general ideas of geography are based on Mercator's maps. As will subsequently be pointed out, this projection has many good qualities for special purposes, and for some general purposes it may be used for areas not very distant from the Equator. No suggestion is therefore made that it should be abolished, or even reduced from its position among first-class projections, but it is most strongly urged that no one should use it without recognising its defects and so guarding against being misled by false appearances.

The great feature of Mercator's projection is that the bearings of all points on the map correspond with the true compass bearings of the routes between them. For instance, on the map San Francisco lies about 2° north of east of Yokohama, and if a steamer leaves Yokohama and sails 2° north of east it will reach San Francisco. Moreover, a compass placed anywhere

on the map suits for the whole map. Thus, if a ruler be laid on the route from Plymouth to Bermuda and rolled carefully along to the compass set out in the Pacific, the true bearing can be read off. It was this property that first secured its popularity, a popularity which it has retained ever since, though it was known even at the time of its invention that the compass-bearing course was not the shortest route on the globe. At that time, however, long ocean voyages were uncommon. Though ships were years away from their port of departure, and though men circumnavigated the globe, they touched frequently at ports if possible.

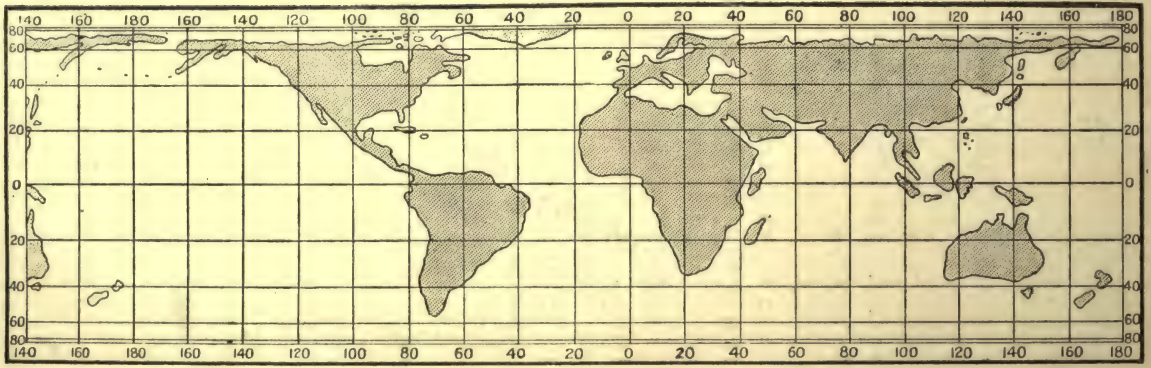
For moderate voyages the compass route agrees well with the shortest route, while for smaller seas, such as the English Channel, the Mercator projection is absolutely perfect, and there is this peculiarity about Mercator maps—viz., that they are all similar. That is to say, if a large number of Mercator charts of various seas covering large or small areas be reduced to such scales that their meridians of longitude are the same distance apart, they will all join up accurately so as to make a flat sheet. Each chart, that is to say, is an exact reproduction of a part of the complete chart enlarged in scale. This is not true of any other projection in general use.

The projection also has the advantage of showing all the inhabited world on one sheet, and it can moreover be prolonged in either direction ; that is to say, it may be repeated. On other projections it is often troublesome to determine the relative positions of two places which appear on different maps which will not

join, such as maps of two hemispheres. Mercator maps sometimes show the 360° of the Equator only, the usual place of division in English maps being in the Pacific Ocean. In this case the relative positions of Yokohama and San Francisco cannot easily be seen, but if a few degrees be repeated, as shown in fig. 13, this difficulty is entirely obviated. The good points of the projection, then, are that compass bearings on the map agree exactly with compass routes on the globe ; that all maps, whether of large or small areas, are similar, differing only in scale ; and that the entire inhabited world can be shown on one sheet, this last property being common to several projections, though it is generally accompanied by great distortion. The bad qualities are that a straight line on the map does not even approximately indicate the shortest route on the globe, and that the exaggeration in size as the pole is approached is so great as to give utterly erroneous ideas of geography in general.

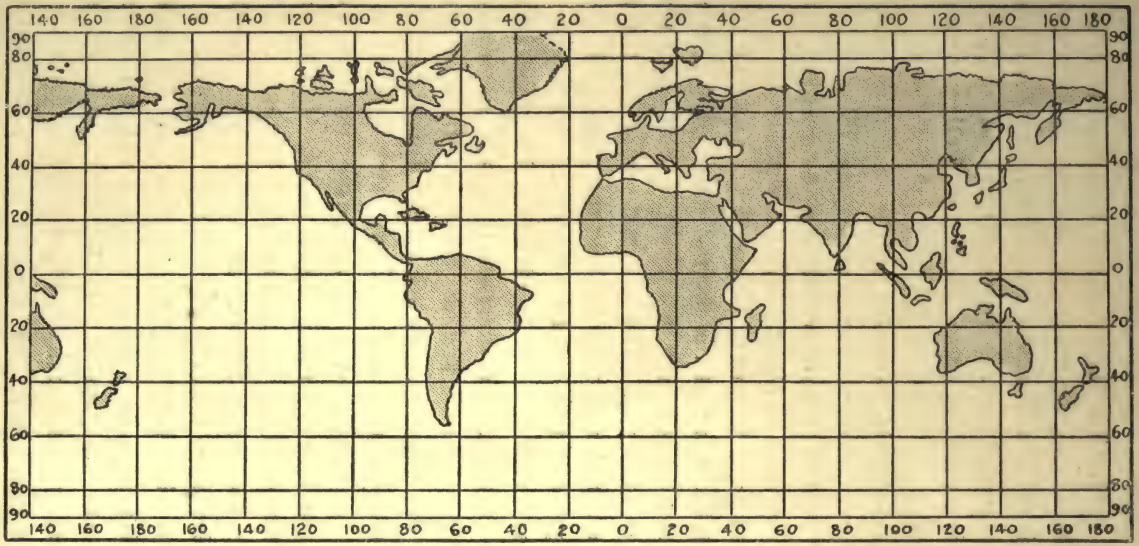
The extent to which polar areas are distorted is well shown by comparing the Mercator map with the map on CYLINDRICAL PROJECTION, fig. 14. This projection, which is very little known, is made by projecting each point on the sphere at right angles to the axis of the sphere out to a circumscribed cylinder touching the earth at the Equator. The surface of such a cylinder has the same area as the sphere, and the projection is an equal area projection ; that is to say, all countries retain their true areas with mathematical accuracy. The distortion of shape near the poles is very great, and the projection cannot be called very generally useful ; still, rightly used, it is instructive.

FIG. 14



CYLINDRICAL PROJECTION

FIG. 14A



EQUIDISTANT CYLINDRICAL PROJECTION, OR PROJECTION BY RECTANGULAR CO-ORDINATES WITH EQUATOR AS CENTRAL GREAT CIRCLE

A projection intermediate between Mercator and cylindrical may be made by plotting the Equator and meridians as in Mercator, and by dividing the meridians into degrees of equal length throughout, and equal to degrees on the Equator. Such a projection is shown in fig. 14A.

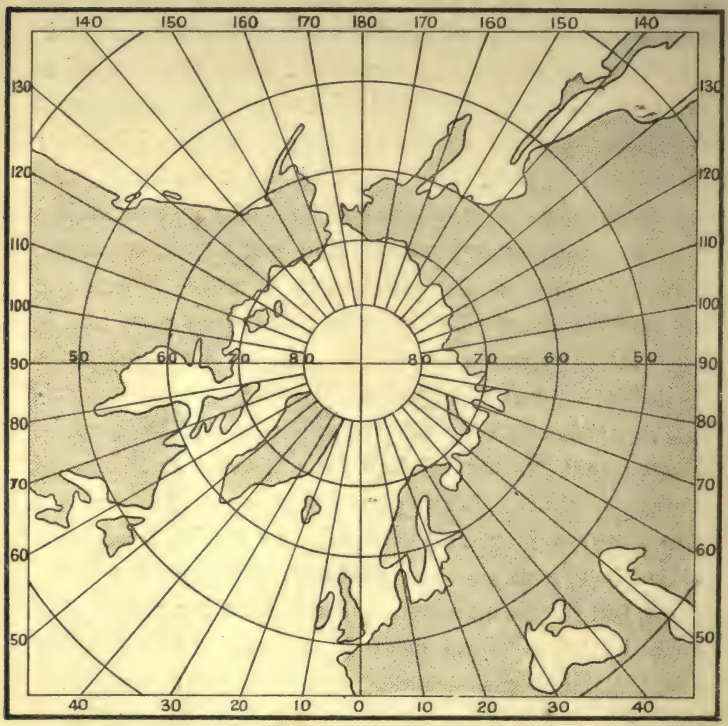
Having said so much on the matter of areas, the question of shortest route remains, and this can be most easily treated of when considering the next projection—viz., *Gnomonic* or *Great Circle Projection.*

The great feature of GNOMONIC or GREAT CIRCLE PROJECTION is that the shortest route on the surface of the globe between any two points is shown by a straight line on the map. The shortest route on the globe between any two points is along the great circle passing through those two points. It will be remembered that a great circle is a circle dividing the globe into two equal portions. To illustrate this, take an orange and assuming the stalk and the mark on the opposite side to be the poles, roughly sketch the Equator and one or two parallels of latitude. Then mark two points anywhere on the orange, and examine how the orange could be cut in halves through those points. If they are on the Equator, it can be cut through the Equator. If they are on the same parallel of latitude and exactly on opposite sides, it can be cut through the poles. If they are on the same parallel, but not so far apart as opposite sides, then the cutting line will run nearer the pole than the parallel. That the line so found is the shortest is easily tested with a piece of thread. It is always possible to cut a globe into two equal parts

through any two points, so that a great circle can always be drawn through any two points, but that great circle can never run along a parallel of latitude, because a parallel is a small circle dividing the globe into two unequal parts. In Mercator's projection, if one point lies exactly east of another point, the direct route on the map appears to run along the parallel of latitude. It is true that by a long and somewhat difficult calculation the great circle can be marked out, and two such great circles or shortest routes are shown on the Mercator map (fig. 13), and tables are published to facilitate such calculations, but this part of the treatise is intended to deal with general impressions and not with navigation, and the point to be considered is this. When the ordinary business man looks at a Mercator map and is told that the route shown by the curved line from Yokohama to San Francisco is some 200 geographical miles shorter than the route shown by the straight line, or that the curved line from London to Shanghai shows a route about 500 miles shorter than the straight one, is it easy for him to realise what this means? He may and no doubt does believe the statement, but he can hardly help thinking of the curved line as indicating a curved route in opposition to a straight one. The expression "great circle" helps to strengthen this view, and it is not uncommon to hear people talk of having taken the straight course across the Pacific (meaning the Mercator course which is sometimes taken to avoid cold weather) when in reality they have taken the curved course. Now, all this goes to prove that people's ideas of geography are not founded on actual

facts, but on Mercator's map. Unless anyone looking at a map fairly grasps the fact that the routes above mentioned running far north are the shortest routes—unless, in fact, it seems to him the natural thing to talk of such routes

FIG. 15

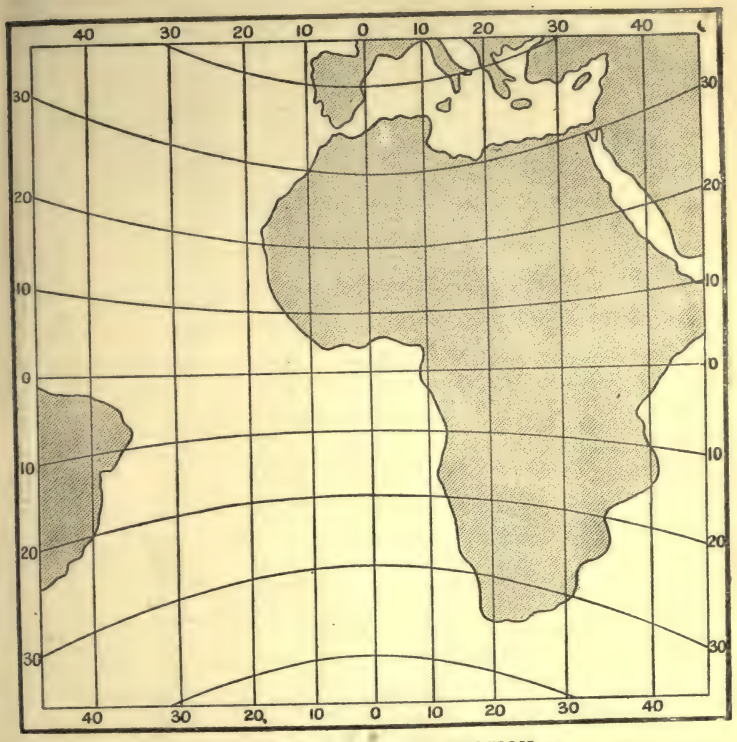


GNOMONIC PROJECTION

as the straight routes, and the routes shown by straight lines on the map as curved routes—his ideas of geography are badly warped, and the sooner he begins to study other projections the better. Maps on the great circle pro-

jection are shown in figs. 15, 16, 17. The principle of construction is as follows : Suppose a plane N D H K (fig. 28) to touch the globe at D. Suppose, as in stereographic projection, knitting-needles be pushed in through the points

FIG. 16



GNOMONIC PROJECTION

F G, but in this case directed to the centre of the earth, and suppose these to be prolonged outwards to touch the plane in H and K. By making a sufficient number of points the latitude and longitude lines can be described.

Let L M be a great circle, then the knitting-needles marking various points of it will be like spokes of a

FIG. 17 GNOMONIC PROJECTION

wheel, and when projected out to the plane they will mark a straight line on it. Even to the non-mathematical mind this will become clear after a little thought.

It thus appears that on a map so constructed all great circles will be straight lines, so that the shortest route between two places can be laid down with a ruler and a pencil. The maps are not good for general purposes, but they are useful to the general public as correctives. They are, however, extremely useful for navigation ; yet, strange to say, no map on this projection is published by the Hydrographic Department of the Admiralty, nor is any map on this projection to be purchased in London except one correct but badly arranged map of the North Atlantic, and British ships sailing on the Pacific have to make use of American great circle maps.

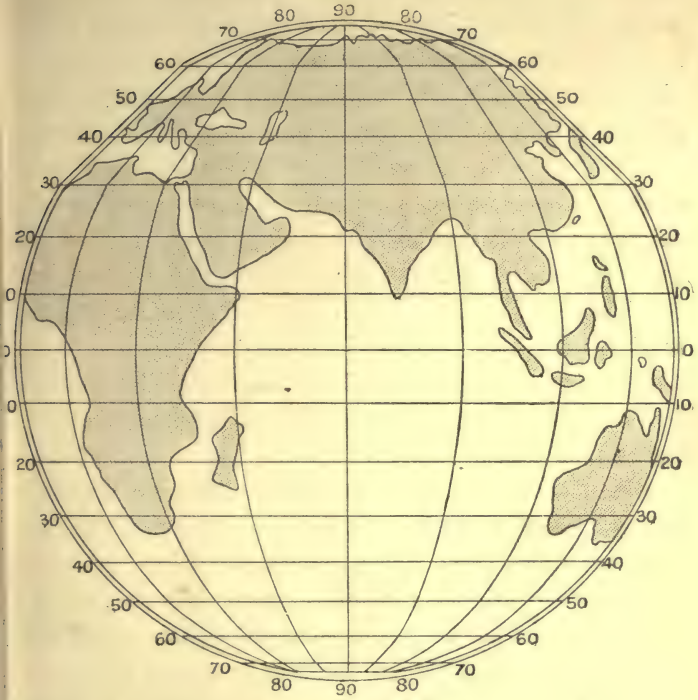
The matter may be made somewhat clearer if by a stretch of the imagination the reader can conceive of two gigantic towers being erected, one at Yokohama and one at San Francisco, with foundation at the bottom and oxygen at the top supplied by Jules Verne. If these two towers be so high that each can be seen from the other, then in imagination it would be possible to anchor buoys at intervals along the route so that they would appear to be in a straight line. Now clearly that line would be the shortest route between the places. It might be a little puzzling at first to find that the bearing of San Francisco from Yokohama was approximately north-east by east, and the bearing of Yokohama from San Francisco approximately north-west by west. Anyone sailing along the route thus buoyed out would therefore begin by sailing north-east by east, when about half-way he would find that he was sailing east, and before arrival he would be sailing south-

east by east. While, moreover, he would start from latitude 35°, and arrive at latitude 38°, he would be at about latitude 47° when half-way over. A compass route would lie between the parallel of 35° and 38° the whole way, and this shows that a compass route is not the shortest route. Another way of considering compass routes is to think of what would happen if one started from any point and continued to travel north-east. It is clear he could never reach the pole, because the pole is north of every place. At the same time he would continue going north ; in other words, his course would be a spiral round the world. Now, a spiral of this sort cannot be a straight line, though any short piece of it may be approximately straight ; consequently a compass-bearing route is not the shortest route except it be due north and south—*i.e.*, on a great circle called a meridian, or east and west at the Equator.

The projections hitherto described have been those suitable for showing large areas, and, as was remarked at the beginning, maps of small areas can be made so nearly correct that no distortion is apparent, and in many cases they are sufficiently correct to have a scale of miles attached. There is, however, one projection which is made use of to a very large extent in atlases for showing maps of countries which have too great an area to be shown with absolute accuracy, but still can be shown so nearly correct that the distortion is not noticeable without careful measurement. It is called CONICAL PROJECTION. Suppose a globe to be made of somewhat porous material like unglazed earthenware, and suppose the lines of latitude and longitude, as also

the countries, to be painted on it, and the colour to sink in a little way, always sinking towards the centre. Then, as in fig. 18, let a band of it be turned down in a lathe, so as to form a portion of a cone. It is evident that

FIG. 18



CONICAL PROJECTION

everything that was on the surface of the sphere will appear on the surface of the cone with no appreciable distortion. Now, the surface of a cone is what is called a developable surface ; that is to say, it can be spread

out flat. A piece of paper, somewhat the shape of a boy's Eton collar, can be cut so that it will lie perfectly flat on the surface of the cone, and the map can be copied exactly on to the paper; the only distortion in the process is that between the sphere and the cone, which, even when the band covers 30° of latitude, is extremely small. The meridians of longitude are straight lines, and the parallels of latitude are circles. Areas and distances are for all practical purposes exact, and shortest routes are very nearly shown by straight lines.

This is such a magnificent projection that it is often employed for areas larger than those for which it is suitable, certain modifications being introduced which generally make the meridians curved lines, but all consideration of such details must be left to the mathematical portion of this treatise.

There is a projection which is not much known but which possesses useful properties, and which may be referred to in this portion of the treatise as ELLIPTICAL PROJECTION (see fig. 19). In this the entire world is shown as an ellipse, the major axis (which represents the Equator going right round the earth) being twice the length of the minor axis (which represents the central meridian from pole to pole—*i.e.*, going half round the earth). The Equator is divided into, say, thirty-six equal parts, the meridians passing through the points of division. The meridian which is 90° east and 90° west of the central meridian is shown by a circle, all other meridians by ellipses, those inside the circle having their major axes north and south, and those outside the circle having their major axes east and west.

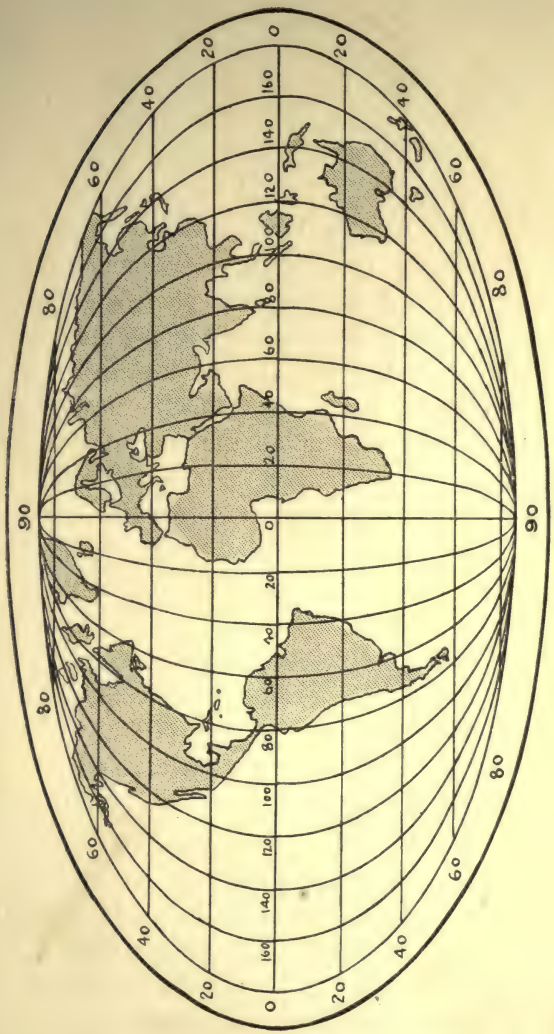


FIG. 19

The parallels of latitude are shown by straight lines, the principles on which their positions are determined being explained later on.

This projection has several advantages. In the first place, it gives a fairly good general idea of the surface of the globe. One can imagine that if drawn on a sheet of indiarubber it could be fitted round a globe of suitable size without very much irregular stretching. In this respect it appears to be much superior to Mercator, or the double hemisphere. In the second place, it is an equal-area projection—that is to say, a map on this projection measuring 14¼ ins. from pole to pole has exactly the same area as the surface of a globe of 10 ins. diameter, and all the countries have mathematically the same area on the map as on such a globe. If then such a map be coloured so as to show the distribution of animals, or of divisions of the human race, or areas of equal temperature, or empires of the world, the ideas gleaned from an inspection of the map as regards relative areas are absolutely correct, while those gleaned from inspection of many other projections are quite erroneous.

Though the foregoing is by no means an exhaustive account of all projections which have from time to time been used, it is hoped that enough has been said to put the general reader on his guard against founding his ideas of geography on maps. The only source of true ideas is a globe, and much good would result from making it the basis of all elementary teaching, warning the scholars that maps are only tolerated for the sake of convenience, and that where map and globe do not

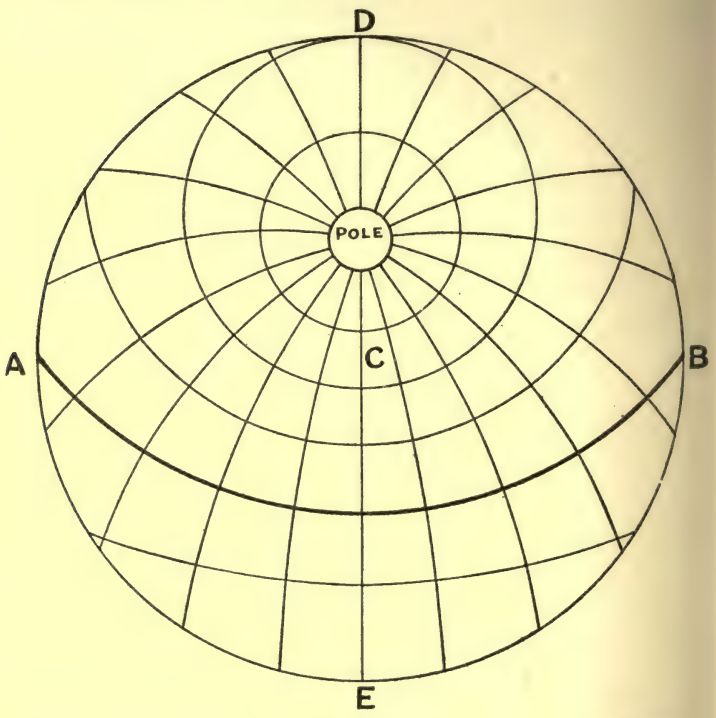
been established that angles are represented by tangents of their halves, it seems simpler to use a table, and the result is better, particularly with large angles.

The meridians clearly all pass through both poles. Now, the South Pole S lies at a distance of $50° + 90° = 140°$ from the centre of the map, and its position is found by laying off the tangent of 70° from the centre. The circle representing the meridian at right angles to the central meridian is clearly symmetrical on both sides of the central meridian—that is to say, its centre is on the central meridian and at F, half-way between the poles. As all meridians pass through the poles, the centres of the circles representing them must be in the line F G or its prolongation, and as the angles they make at S and P are the same as the longitudes of the meridians, if the line S H be drawn for any longitude, and the line S K be drawn at right angles to it, the centre of the circle representing the corresponding meridian will be at K, the distance F K being the co-tangent of the latitude; this peculiar relation holding true although F K is not the Equator, but is the projection of the parallel of latitude 50° south, which passes through the point of sight and hence becomes projected as a straight line. Having found the centres of all the meridians, describe the circles through the two poles, continuing them through the North Pole to the boundary of the map. Fig. 26 shows a map constructed on this principle, the lines being drawn at intervals of 20°.

It is sometimes stated that in the case of a projection with some point such as London for its centre, the bearing and distance of any point from London can be

found by drawing a line from London to that point, measuring the angle it makes with the central meridian and measuring the distance with a stereographic scale —*i.e.*, a scale of tangents of half angles. The young student may be easily misled by this statement. If he

FIG. 26



draws a line from London to Madras he will find that the latter lies almost exactly due east of the former, while in ordinary parlance all points in Europe or Asia on the parallel of London are considered to lie due east of it, while it is considered to lie due west of all of them.

It is true that if a high tower in Madras could be seen from a high tower in London, or if a straight hole could be bored from London to Madras, the direction of Madras as seen over the surface or through the hole would appear to be due east, but London would certainly not appear to lie due west if seen from Madras.

What really is measured is the angle made between the central meridian and the great circle passing through London and Madras, and the distance given by scale is the distance measured on the great circle.

Suppose a vertical rod to be erected in London, and another vertical rod a short distance east of London, and suppose then, without reference to compass bearings, that line is constantly extended by planting rods in the same line as the previous rods, that line will eventually reach Madras. This being the case, it seems right to say Madras is east of London ; still, this is employing the word east in an unusual sense, the common meaning being probably founded on one's ideas derived from the constant use of Mercator maps.

The whole thing will become quite clear to the student as soon as he fully realises that compass bearings are always measured on the horizontal plane at the point of sight, and are not reciprocal, though when the distance between two points is short the difference is trifling.

In surveying, to avoid confusion, the word azimuth is used to indicate the angle which a great circle passing through two points makes with the meridian at the point of sight. Naturally the angle will differ according to

which of the two points is taken as the point of sight, because meridians are not parallel but converge towards the pole.

Stereographic projection has been thus fully treated because it is a projection which can easily be drawn with great accuracy with any point as centre, and hence is useful as a stepping-stone to other projections.

The EQUAL AREA PROJECTION to be now described is that known as Lambert's Zenithal equal-area projection. The diagram (fig. 10) and the map (fig. 9) given in the first part of this treatise will serve for this part also. The circles of latitude in the polar projection are described with radii equal to the chords of the angles measured from the pole or the chords of the co-latitudes.

The circle from which the chords in the figure are taken was chosen of such a size that the chord of 90° should be equal to the radius of the circle used in stereographic projection, so that the maps should be of the same size, but clearly this is merely a matter of convenience. The surface of a sphere is equal to the surface of the circumscribed cylinder, a band of any height on the cylinder being equal to a band of the same height on the globe. That is to say, the surface of a sphere is $4\pi R^2$. Then, supposing the cylinder to surround the Equator, for any angular distance measured from the pole called A, the area of the corresponding band in the cylinder is $2\pi R^2 (1 - \cos A)$.

This much it is assumed is known by every student of this treatise. Now, the chord of any angle A is equal to $2R \sin \frac{1}{2} A$, and the area of a circle of which this is the radius is $\pi (2R \sin \frac{1}{2} A)^2$, or $\pi 4R^2 \sin^2 \frac{1}{2} A$, or

$2\pi R^2 2 \sin^2 \frac{1}{2} A$. In any elementary book on trigonometry there will be found a table of equivalent formulæ, and it will there be seen that $2 \sin^2 \frac{1}{2} A = 1 - \cos A$, a proof of course being given somewhere in the book. Consequently the area of any circle of which the chord of A is the radius, is equal to the area of the sphere enclosed in the parallel whose angular distance from the pole is A. All the quadrilateral figures, therefore, formed by the lines of latitude and longitude on the map are with mathematical accuracy of the same area as the corresponding areas on a sphere the radius of which bears to the radius of the map the proportion of 1 to $\sqrt{2}$.

The chord of an angle being equal to twice the sine of half the angle, the projection can be made by taking a table of sines, laying off the sines of $\frac{1}{2}°$, 1°, $1\frac{1}{2}°$, 2°, &c., up to 45°, describing circles through them and numbering them as complete degrees 1 to 90 (though the sines count from the pole, the numbers according to custom run from the Equator).

To make a projection on any other plane, make some easy projection, such as stereographic, on the required plane, and place a tracing of a polar projection of the same sort over it. Then construct a polar projection of equal areas in pencil, and, noting where the lines of latitude and longitude on the stereographic projection cut the lines of its own polar projection, tick off the corresponding points on the equal area polar projection. Draw the lines of latitude and longitude through these points, rub out the pencil lines, and the work is done. This method may be used for transforming other

projections, and stereographic is generally the best to take as a standard, but the equal area map (fig. 9 A) was transformed from the gnomonic map (fig. 17). The direct projection of equal areas on the plane of London is hardly possible by a direct graphic method.

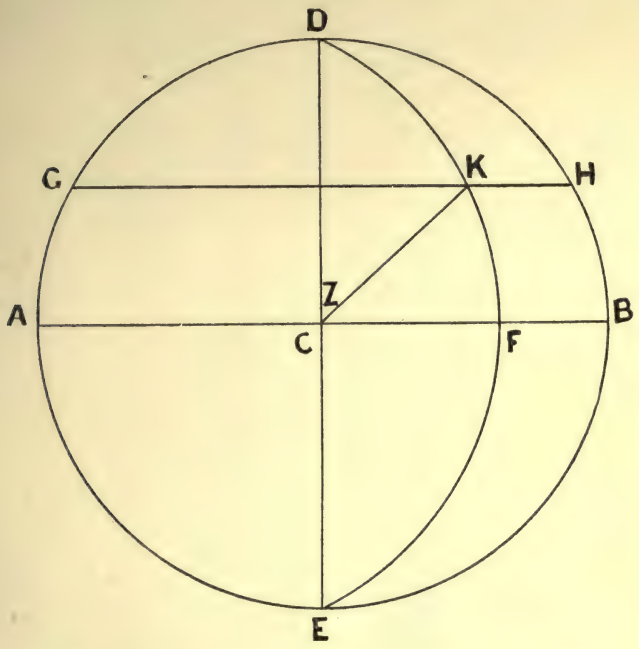
There is naturally another way of making a projection of equal areas on a map bisected by the Equator, and it will probably be convenient to treat this as part of a larger question.

The whole of the projections hitherto mentioned as well as some of the following, such as gnomonic, and the whole series of so-called globular projections, are what are called Zenithal projections—that is to say, lines radiating from the centre of the map represent great circles on the sphere, crossing each other at that point at the same angles as the radiating lines. This is quite clear where the pole is the centre of the map, and great circles may be supposed to exist passing through any other point. Again, degrees on these great circles are represented by some definite scale on the map, such as a scale of sines, or a scale of tangents of half angles, &c. but, whatever be the scale, it is uniform all round; consequently if one knows the position of a great circle passing through the centre point of the map and any given point, and the number of the degrees between the centre and that point, then that point can be laid down on any projection by using the proper scale.

On fig. 27 let A B be the Equator, E and D the poles, E F D a meridian whose longitude is t, and G H a parallel of latitude whose latitude is l. Let them cut each other in K Now, it is quite clear that if the angle

D C K, which may be called Z, is known, and the number of degrees on the sphere between C and K, which may be called θ, then the point K can be plotted on any projection bisected by the Equator by using the proper scale; and by finding a similar line for the crossings

FIG. 27



of a sufficient number of meridians and parallels the entire projection can be made. In all the Zenithal projections the angle Z on the map will be the same as on the sphere, while the scale to be used for the length C K or the angle θ will depend on the projection.

For orthographic projections a scale of sines must

be used; for equal areas a scale of sines of half angles; for gnomonic a scale of tangents; for stereographic a scale of tangents of half angles; for globular a scale found by dividing one quadrant graphically, and drawing lines to the point of sight through the plane of projection, where the scale will thus be constructed.

The formulæ are extremely simple, viz.:

$$\text{Tan } z = \sin t \cot l$$

$$\text{Cos } \theta = \cos t \cos l$$

and tables for every tenth degree are given in Germain's Treatise on Map Projections, but it appears to be impossible to explain these without actually going into spherical trigonometry.

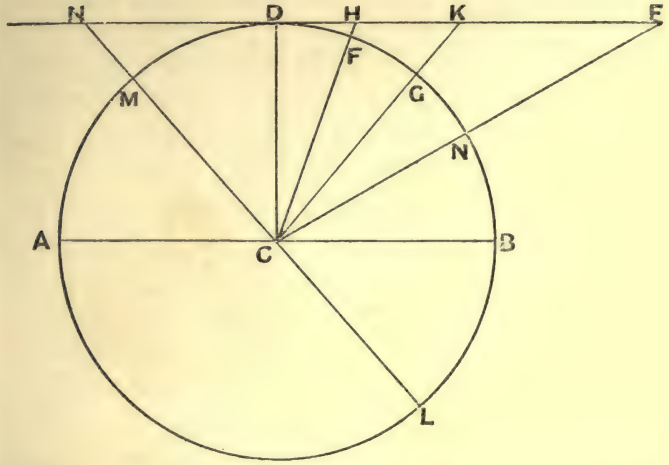
While, however, the detail cannot be explained, it is evident that the system is of general application to transformations. If a stereographic projection be made on any plane, and if a line be drawn from the centre of the map through any point and prolonged to a scale of degrees round the map, if the distance of this point from the centre be measured with a stereographic scale —*i.e.*, a scale of tangents of half angles—and if a line at a corresponding angle be laid down on any other projection, and the point required be plotted with the scale belonging to that projection, it will be plotted in its true place.

This is manifestly in principle the same as applying a tracing of a polar projection as above described, but in practice it has the advantage of enabling the crossings of meridians and parallels to be plotted directly, and so gives a better result.

Instead of following the order of the first part of the treatise, the next projection to be considered will be GNOMONIC or GREAT CIRCLE PROJECTION. In this projection all points on the surface are projected on to a plane touching the surface, by lines drawn from the centre of the sphere; thus the points F G (see fig. 28) are projected on to the plane at H K.

To make a polar projection, draw the meridians

FIG. 28



radiating at equal angles as before, and lay down a scale of tangents from the centre, as far as convenient, and describe circles through them to represent parallels of latitude. The radii of the parallels, that is to say, are the tangents of the co-latitudes. After 60° from the pole this projection is much distorted, and a projection of half the globe is of course impossible. The special property of this projection is that a straight line

between any two points on the map is the projection of the great circle passing through those places, and is consequently the shortest route on the surface of the earth. To be of any real use in navigation maps must have every degree shown, every tenth degree being shown by a thicker line.

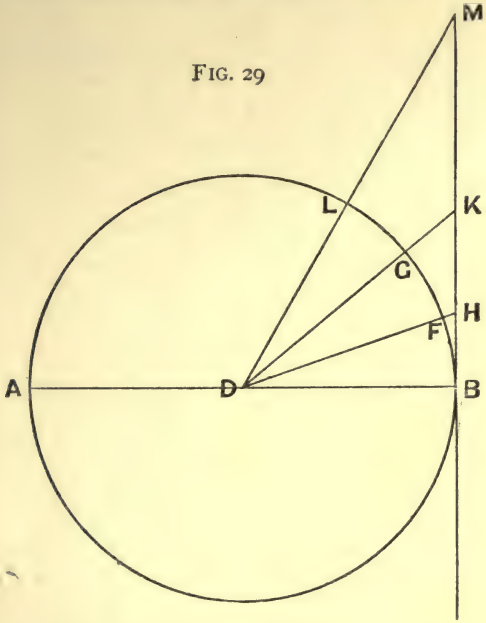
The fact that a great circle on the globe is a straight line is easily seen when it is considered that a great circle is made by a plane passing through the centre of the sphere, and all points on the great circle are consequently projected out along that plane like spokes of a wheel, and the intersection of one plane with another plane is a straight line. These maps are useful for navigation although not very convenient sailing maps. The best way of defining and using a great circle route, or a route approximating to a great circle but avoiding extreme latitudes on account of ice &c., is to lay it down on this map and transfer it to a Mercator. No tables can be made to do this in the same time, because by this map, islands, continents, ice regions, &c., can be avoided at once, whereas by tables the mathematical route must be laid down and afterwards modified. The route to avoid an extreme latitude is found on these maps by drawing straight lines from the two ports touching the limiting parallel, which can be done in a moment, while it takes some little time to lay it down on a Mercator.

A set of maps which show the whole globe will, if fitted together, form a solid ; hence the smallest possible number of maps is four, each of which, if all are the same size, would be an equilateral triangle. It is usual, however, to have six square maps, two being polar and four

equatorial maps, which form a cube. Six maps, each measuring 120° across, will, so to speak, overlap, but will form useful maps. If only 90° across they have no overlap and are not very useful.

For practical purposes, however, maps should be made to take in certain areas, such as the North Pacific, and

FIG. 29

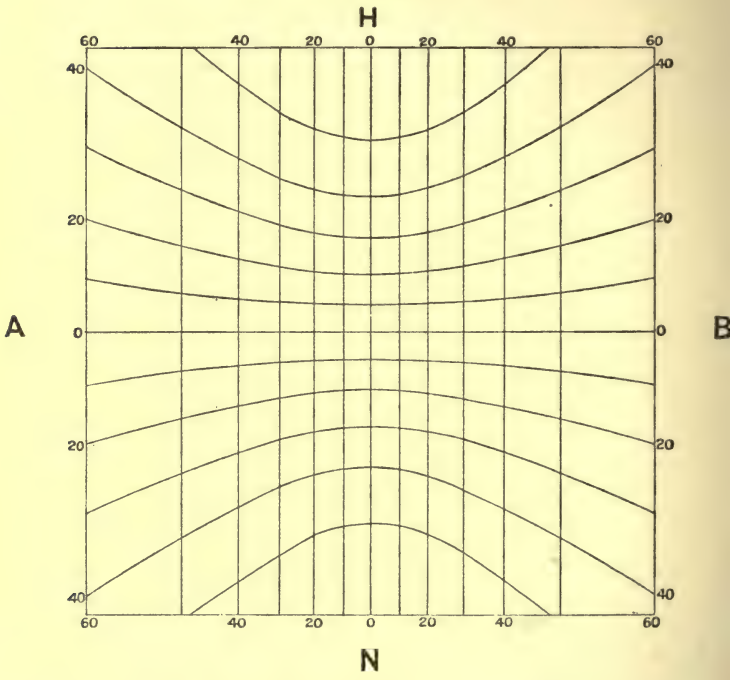


in such cases it is often convenient to make the plane touch at a point neither on the Equator nor on the pole, but generally at a point in 45° lat.

To make an equatorial map, let fig. 29 be a diagram looking down at the North Pole D. The circle then will show the Equator. Mark equal angles round as at F G and project them to H K. Tick off the lengths, B H,

B K, &c., and transfer them to the Equator A B on fig. 30, or, from a table of natural tangents lay down the tangent of each degree and draw the meridians through these points at right angles to the Equator. It is clear that on diagram 29, if the quadrant which is represented by the

FIG. 30



line B D were divided into degrees, and radii were drawn through the points of division and projected to the plane of the map, they would cut the plane at distances from the Equator equal to the natural tangents of the angles. The central meridian represented by D B on fig. 29 and

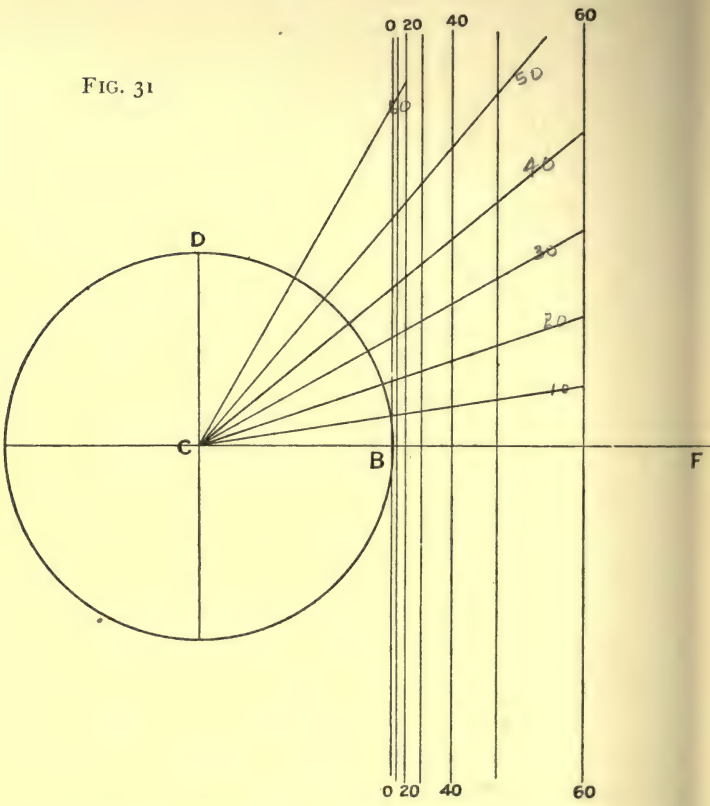
by N H on the map may therefore be divided on the map in the same manner as the Equator—that is to say, the degrees may be laid down from a table of natural tangents to radius D B, and any other meridian represented, say, by D F on fig. 29, and by the meridian of 20° on the map, may be divided on the same principle—that is to say, by a scale of tangents, but they will be tangents to radius D H and not to radius D F. Now, D H is the secant of the angle B D H, so that on each meridian the degrees of latitude are laid down from a scale of tangents to a radius equal to the secant of the angle of longitude between the central meridian and the meridian in question. The graphic solution is very simple.

Having set out the meridians from a scale of tangents, draw the diagram (fig. 31) in which C B is equal to the radius of the sphere. Lay off, along the line C F, the natural secants to radius C B of each degree as far as it is intended to extend the map, say to 60°. Draw lines at right angles to C F, through these points, then draw radii for each degree cutting these lines. Tick off the intersections for each meridian and transfer them to the map, then draw the parallels of latitude through them. The lines will be hyperbolas, but can easily be drawn in by a good draughtsman from the points of intersection without reference to the particular form of the curve. The whole construction is exceedingly simple, but the work is considerable as the lines must be carefully drawn at each degree and a very slight irregularity will be noticeable.

Where the plane touches at 45°, the diagram of construction becomes as in fig. 32, and the map appears as

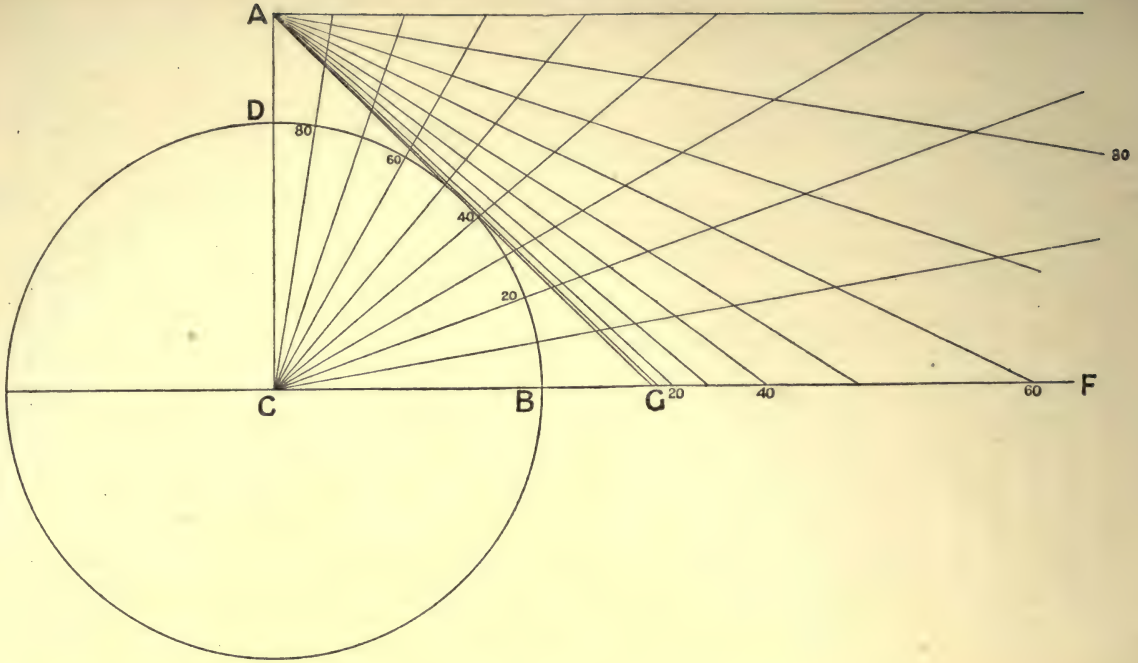
in fig. 34. The method of construction is as follows: Consider fig. 33, which is a plan looking down on the pole and showing the map F L M K. It is clear that the line K F is shown of the same size as on the map,

FIG. 31



and the meridians cut it at distances from G equal to the natural tangents of the angles of longitude from G to radius D G. The meridians cutting the line L F as shown in plan also cut at distances from L equal to the natural

FIG. 32

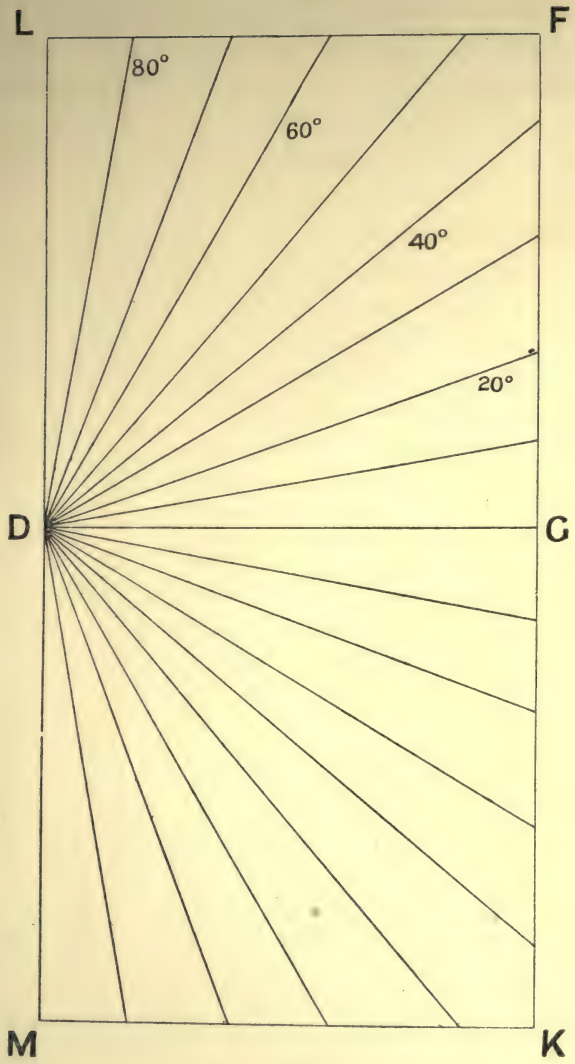


tangents from L to radius D L *as they appear on the plane of the Equator in fig.* 33, but in transferring them to the map the length must be increased in the proportion of L F on fig. 33 to L F on fig. 34. All the meridians can then be drawn on the map from the point A to the points ticked off on the border line. In fig. 32 draw A G at 45° to A C and C G. Then, along C F, mark off secants of each degree to radius C G (not to radius C B). Draw lines from A to the points thus found. Draw radii from C for each degree on the quadrant D B, producing them to cut the lines drawn from A. Tick off the distance from A along the several meridians on slips of paper and transfer them to the corresponding meridians on the map. Draw curves through the points, and the work is done. In this case all parallels north of 45° will be ellipses, 45° will be a parabola, and all south of 45° hyperbolas, but in practical construction a sufficient number of points must simply be ticked off to enable the curves to be drawn without reference to their character.

One difficulty arises out of the fact that the secant of 90° is infinity, so that one cannot plot the secant of 90° or indeed of any very large angles. This may be got over by plotting versed lines of the longitude to radius C G upwards from G. It is believed that this simple construction of gnomonic maps has not hitherto been published. At all events it is not generally known, and indeed in many treatises but scant attention is given to this exceedingly important projection.

The four projections hitherto described may be compared by reference to the radii of the circles showing the parallels on polar projections, counting the angles from the poles or the co-latitudes.

FIG. 33



Orthographic .	. Sines of the angles.
Equal areas .	. Sines of half the angles.
Gnomonic . .	. Tangents of the angles.
Stereographic .	. Tangents of half the angles.

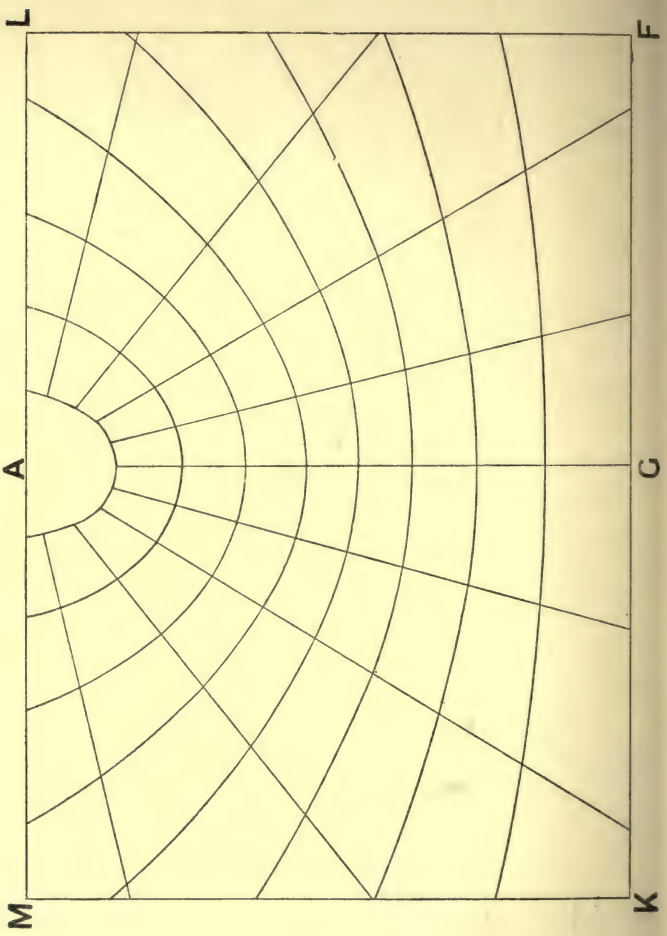


FIG. 34

As regards the area that can be shown :

Orthographic . .	One hemisphere.
Equal areas . .	The whole world.
Gnomonic . .	Anything less than one hemisphere.
Stereographic . .	Anything less than the whole world.

It will here be convenient to treat that system of projections which may be included in the general (though somewhat unsuitable) name of GLOBULAR.

It has been seen that when the earth is viewed from an infinite distance—*i.e.*, in orthographic projection—the outer portions of the hemisphere are unduly reduced in size in proportion to the central portions, while when the point of sight is at the surface of the sphere—*i.e.*, in stereographic projection—the outer portions of the hemisphere are unduly enlarged. It must be possible, therefore, to choose some point intermediate between them where the general result is more satisfactory. In fig. 35 let A H be an angle of 45° and let $C M = \frac{C F}{2}$. Join H M and produce it to D. The length B D will be found by measurement or calculation to be $R\frac{\sqrt{2}}{2}$. If this point be chosen as the point of sight and lines be drawn to G and K, representing points $22\frac{1}{2}°$ and $67\frac{1}{2}°$ respectively from A, then the lines C L and C N will be found to be ·25 and ·75 of C F, correct to two places of decimals, though in the third place there is a discrepancy ; and if a hemisphere of the globe as seen from D be projected on the plane represented by F E, the result will be generally very

satisfactory, and a good general idea of half the globe will be obtained from the map. This projection is known as La Hires, though it is not clear why the name

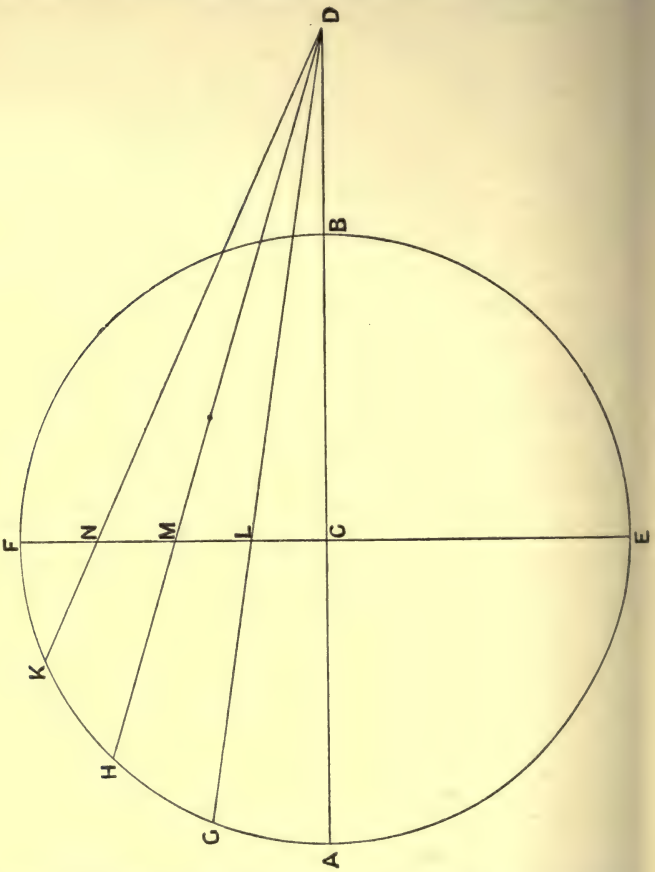


FIG. 35

of any individual should be attached to such a self-evident system. If A be the pole, the projection is very easily constructed, as the meridians radiate in the usual manner, and the parallels are described with radii found by tick-

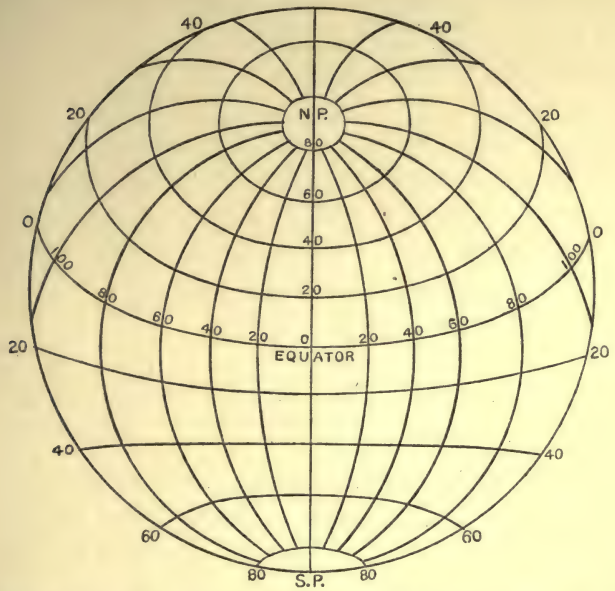
ing off the points on the line C F. The projection of a hemisphere so drawn is undistinguishable, except by the most careful measurement, from one where the parallels are drawn at equal intervals as in fig. 11. If E and F be the poles, the construction graphically is very difficult, while if A is situated neither on the pole nor on the Equator it is more difficult still, but it can be transformed from a stereographic projection by the method of radial measurement, using a stereographic scale on the stereographic map, and the scale ticked off from C F on the globular map, but the result will be to all appearance the same as if an ordinary scale of equal divisions were used. It will also be approximately the same as the projection shown in fig. 12, where the Equator, the central meridian, and the circumference are each subdivided into equal divisions, and the meridians and parallels are all circles. The difference between the two, though slight, is quite perceptible to the eye, but the general appearance of fig. 12 is more pleasing than that of the globular projection above described, and on that account in the author's opinion preferable to it, as for all practical purposes the two are equally correct. There is no doubt some force in the argument that the projection shown on fig. 12 is hardly entitled to the term projection at all, as it is not constructed on any true principle, nor even, as some projections are, on one arbitrary principle throughout, but each portion of it is constructed independently in an arbitrary manner, and it cannot therefore be transformed from or into any other projection ; still, it forms the basis of a very useful and pleasing map.

If projections of the hemisphere only had to be considered, this series of projections might now be dismissed, but two cases must be referred to. While in orthographic projection no more than one hemisphere can be shown, and in stereographic anything less than the whole globe can be shown, in globular projection so much of the globe can be shown as lies beyond the touching points of the tangents drawn from the point of sight or the touching circle of the tangential cone. For certain purposes it is useful to show more than half the sphere, and when this is the case the arbitrary projection above referred to entirely breaks down. Again, when a country or continent of a considerable size has to be shown, so large that the projection chosen will be of importance, yet not so large that the ineradicable errors are so great as to render attention to details superfluous, a globular projection is supposed to present some advantages.

The general case has been investigated by Airy and others, but in the most complete manner by Colonel James and Colonel Clarke. For a projection showing areas of about $\frac{2}{3}$ of the sphere, each adopts a globular projection, James placing the point of sight at a distance from the centre of the globe equal to radius $\times 1{\cdot}5$, while Clarke adopts a distance of radius $\times 1{\cdot}367$. In the case of a map on the scale shown in fig. 36 this makes no appreciable difference, but in a larger scale the difference is distinct and is in favour of Clarke. As will be seen by an inspection of fig. 36, which shows a portion of the sphere of which the spherical radius is $113\frac{1}{2}^\circ$ and the centre point in latitude 23° 30′ the projection is highly satisfactory, both as regards general appearance and

general accuracy. For smaller projections Colonel Clarke considers that the distance of the point of sight should depend upon the area to be shown. The following table gives his figures for the distance of the point of

FIG. 36



sight from the centre of the globe, depending on the number of degrees of the sphere between the centre of the map and the edges, or, as he puts it, the spherical radius of the map.

Spherical radius of map	Distance of point of sight from centre of sphere in terms of radius of sphere
40°	1·625
54°	1·61
90°	1·47
108°	1·40
113½°	1·367

A map of Africa can be plotted on a projection with a spherical radius of 40°, and such a projection is shown on fig. 37. There is also shown on fig. 37A the central portion of the arbitrary projection, of which a hemisphere is shown on fig. 12, the extreme length of Equator and central meridian being kept the same in figures 37 and 37A for the purpose of comparison. On the scale of these diagrams the difference is not easily seen, but the co-ordinates for plotting given by Colonel Clarke will show the special properties of his projection.

The first thing that strikes one in looking at these co-ordinates is that the Equator and the central meridian are divided similarly. This shows that the projection is that of a sphere and not of a spheroid.

The dimensions are as follows:

	0°	10°	20°	30°	40°
Clarke .	0	9·69	19·43	29·25	39·17
Arbitrary	0	9·79	19·58	29·38	39·17

The difference is distinct though not very great, but it has to be borne in mind that the divisions are equal on the globe, and that this similarity between globe and projection is sacrificed in Clarke's projection.

The co-ordinates of the crossing of 40° latitude and 40° longitude are as follows:

Clarke . . .	32·44	42·34
Arbitrary . .	31·14	42·82

The lengths of parallel and meridian on the globe (which approximately correspond with the co-ordinates) are 30·00 and 39·17.

Neither of the projections is here very close to truth,

FIG. 37

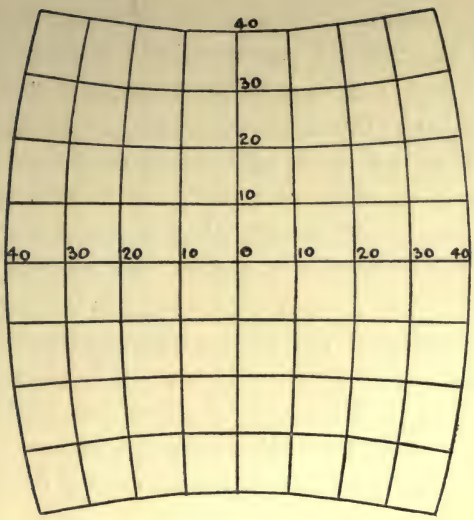
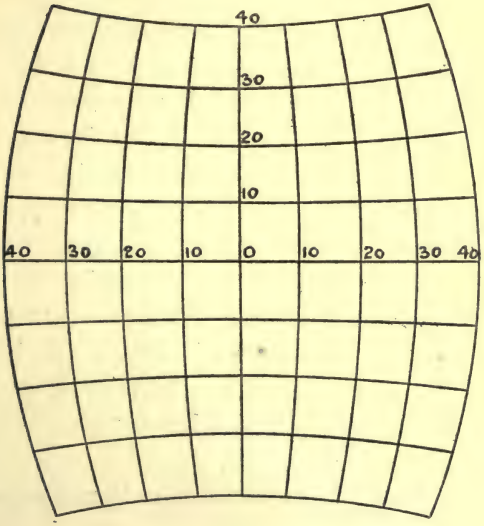


FIG. 37A.



and it is, of course, clear that the excess of length on meridian and parallel corresponds to a certain extent with the excess of length in Clarke's projection of the division on the Equator from 30° to 40° over that from 0° to 10°, thus helping to preserve local similarity of shape, a point made much of in the case of Mercator and Stereographic projection, but in the author's opinion not only useless, but misleading. It should also be noted that when one gives lengths of 10° in three significant figures the spheroidal form of the earth might be considered. If the length of 40° on the Equator is 39·17, the length of 40° of the meridian on the spheroid is 38·96, a difference 50 per cent. greater than the maximum difference between any of the globular and the arbitrary divisions.

As was said at the beginning of this treatise, maps may be made to look as nearly like the globe as possible, or they may be made to retain some special quality at the expense of others. When this special quality is actually made use of no pains must be spared to retain it in perfection; where this is not done a projection on mathematical principles is not a bit better than one on arbitrary lines, while the former may be misleading by inducing people to suppose that the projection is more correct than it really is. In the case of a map of Africa the author is inclined to give the preference to the arbitrary projection, which is almost perfect over all the continent except a small portion of the north-west coast. The other inaccurate portions are in Arabia, or in the ocean.

The ELLIPTICAL PROJECTION, to be now considered,

called the equal-area projection of Mollweide, is at once ingenious and very satisfactory for general purposes. Its advantages have been referred to in the first part of the treatise.

To construct it let A B (fig. 38) represent the Equator, and D E = half A B represent the central meridian, the two lines bisecting each other and being at right angles. From C as centre and C D as radius, describe a circle; this will represent the meridian 90° each way from E D. Through D B E A describe an ellipse; this will represent the central meridian on the other side of the globe. Divide A B into 360 equal parts to represent degrees, or 36 to represent spaces of 10°. Through the points of division describe ellipses which pass also through D and E. Let D H E, D K E, be two of these ellipses. Now, if one axis of an ellipse is the same as the diameter of a circle, the area of that ellipse bears to the area of the circle the same proportion that the other axis of the ellipse bears to the diameter of the circle. Hence the area included between D E and the semi-ellipse D H E bears to the area of the half circle contained between D E and D G E the same proportion that C H bears to C G, and consequently the area contained between D H E and D G E bears to the area of the half circle the same proportion that H G does to C G. Therefore, if ellipses be drawn through all the points of division, the figures contained between two arcs of adjacent ellipses and the Equator will be equal to each other, and equal to the corresponding surfaces on a globe whose diameter is equal to $\frac{DE}{\sqrt{2}}$ as proved earlier.

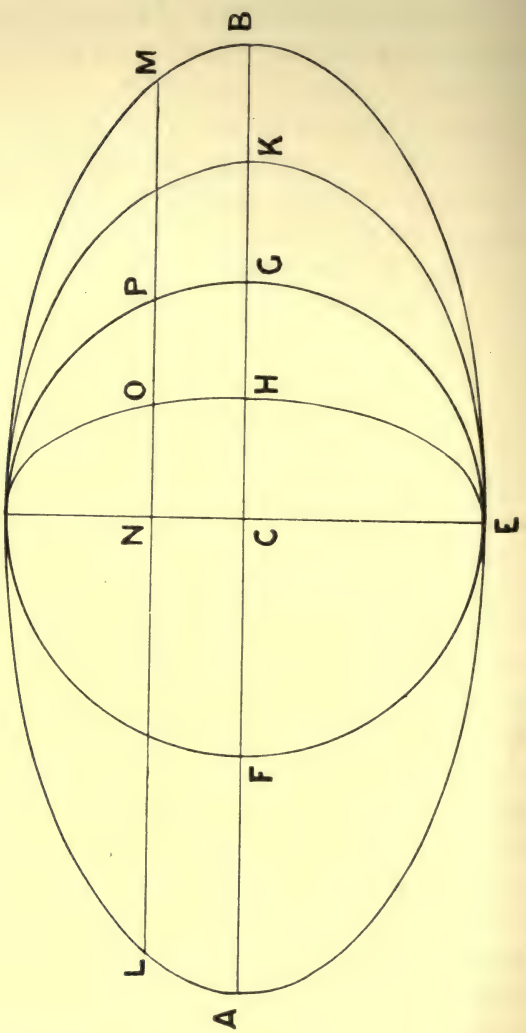


FIG. 38

If a line L M be drawn parallel to A B, the portion N O H C cut off the ellipse will bear the same relation to the half ellipse as N P G C bears to the half circle, and consequently lines drawn parallel to the Equator will cut off equal portions of all the spaces enclosed between adjacent ellipses, and if they be placed at proper intervals they will represent parallels on an equal area projection. As all meridians are affected equally, consider the case of the meridian 90° east and west of the central meridian. This is a circle, and in order that a parallel may be correctly drawn it must divide the circle in the same proportion that the corresponding parallel divides the surface of the sphere. It has been seen that the surface of the sphere enclosed between any parallel and the Equator is proportional to the sine of the latitude of the parallel—thus the parallel of 30° divides the surface of the half sphere into two equal portions, therefore the line on the projection to represent it must divide the half circle into two equal portions. No easy method can be given for calculating this directly, but the inverse problem is simple in principle. In fig. 39 let A B be the Equator and D E a line representing a parallel. The area of the segment D F E G is equal to the area of the sector D C E G less the triangle D C E, both of which are easily calculated if C F is known.

Taking radius as unity, let C F=·4000.

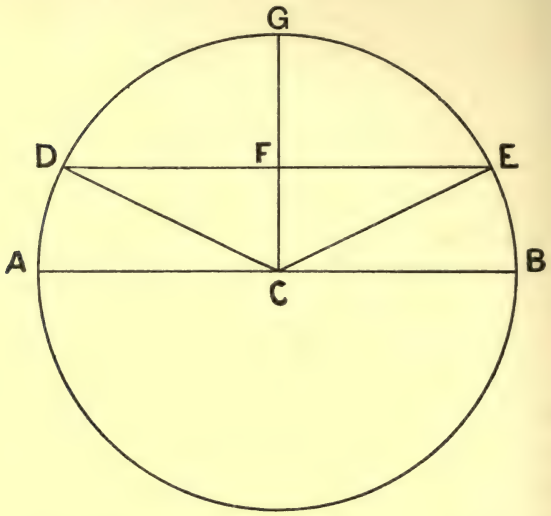
This will make the angle F C E=66° 25′, and the line F E=·9165.

The circular measure of 66° 25′ in terms of radius is 1·1592, and 1·1592 is therefore the area of the sector D C E G. The area of the triangle D C E is ·9165 × ·4000

=·3666. Deducting this from 1·1592, the area of the segment is found to be ·7926.

If now the length C F be assumed to be ·4050, the area of the segment will be found to be ·7835, and by interpolation a length of ·404 will give the segment an area of ·7854, or half the half circle. Hence the

FIG. 39



parallel of 30° latitude lies at a distance of ·404 from the Equator.

By assuming a sufficient number of lengths, and interpolating, the lengths corresponding to each degree or each 10° can be found.

The following table extracted from a large table in Germain's Treatise gives the distance from the Equator of each parallel at intervals of 5°.

TABLE FOR THE CONSTRUCTION OF ELLIPTICAL MAP ON THE SYSTEM OF MOLLWEIDE AND BABINET. DISTANCE OF PARALLELS OF LATITUDE FROM EQUATOR, RADIUS BEING UNITY

Latitude	Distance	Latitude	Distance
°		°	
5	·06851	50	·65116
10	·13681	55	·70804
15	·20472	60	·76239
20	·27202	65	·81382
25	·33851	70	·86191
30	·40397	75	·90606
35	·46820	80	·94540
40	·53097	85	·97838
45	·59204	90	1·00000

The next projection to be considered is one of an entirely different character—viz., MERCATOR'S. Those previously described have been capable of graphic construction, and four of them have been perspectives of the globe, one from an infinite distance, one from a short distance, one from a point on the surface, and one from the centre. The projection of equal areas (fig. 9) does not represent any view of the earth, though it looks a little like some of the perspectives. In the case of Mercator no graphic construction is possible, though it somewhat resembles a map made by projecting all points in straight lines from the centre to touch a circumscribed cylinder.

On a sphere the degrees of longitude decrease from the Equator to the poles, and in order that the map of any area should agree in shape with the actual area, the proportion of length of degrees on the meridian and parallel must be kept the same as on the globe.

In Mercator's projection (see fig. 13) a straight line is taken to represent the Equator and is divided into 360

equal parts. Lines are drawn through the points of division at right angles to the Equator. These represent the meridians. Seeing, then, that the degrees of longitude on the map remain constant throughout, the only way to keep up the proportion is to increase the size of the degrees of latitude from the Equator towards the poles.

Thus at 60° of latitude on the sphere a degree of latitude is twice as long as a degree of longitude, but as in Mercator's projection the degrees of longitude are the same everywhere, the degree of latitude at 60° latitude must be made twice as long as a degree of latitude at the Equator.

The following explains the principles on which a Mercator map may be made.

On the sphere the length of a degree of longitude at any latitude is equal to the length of a degree of longitude at the Equator multiplied by the cosine of the latitude, or, what is the same thing, divided by the secant of the latitude.

Now, on Mercator the length of a degree of longitude at any latitude is equal to the length of a degree of longitude at the Equator—that is to say, it is equal to the corresponding degree of longitude on the sphere multiplied by the secant of the latitude, and, as the true proportion of length is to be maintained, every degree of latitude on the sphere must be shown on Mercator by a length equal to itself multiplied by the secant of its latitude.

As one geographical mile or one minute of arc is taken as unity, each minute of latitude on the sphere is

represented on Mercator by its own secant. If, then, the variation of the secant in intervals of one minute be neglected, a table of lengths of minutes of latitude on Mercator, or 'Meridional Parts,' may be made by consecutively adding the secants of 1′, 2′, 3′, 4′, &c., and as a matter of fact for moderate angles this table will agree with tables given in books.

In the case of large angles, however, the variation of the secant even in intervals of one minute becomes important and has to be allowed for.

Now, it was discovered long ago that the meridional parts corresponding to latitude were proportional to the logarithmic tangents of $45° + \frac{L}{2}$, where L is the latitude, and the formula for calculating them became merid. parts $= R \log \tan (45° + \frac{L}{2})$.

To put this into workable shape R has to be replaced by the length of radius expressed in minutes of arc, or 3437·7468, and if common logarithms are to be used, it is necessary to deduct 10 from the tabular number and to divide by the modulus of common logarithms, or ·4342945.

Now, $\frac{3437{\cdot}7468}{{\cdot}4342945} = 7915{\cdot}7046$, and the formula becomes

$$\text{merid. parts} = 7915{\cdot}7046 \left\{\log \tan (45° + \tfrac{L}{2}) - 10\right\}$$

It is impossible to grasp the rationale of this without what in this treatise must be called higher mathematics, but the table based on secants should be clear to anyone.

The following table based on the logarithmic

formula given above gives the meridional parts for every 10°. Complete tables are given in Chambers's and all other educational sets of mathematical tables.

TABLE OF MERIDIONAL PARTS ON THE SUPPOSITION THAT THE EARTH IS A SPHERE

Latitude	Merid. parts	Latitude	Merid. parts
°		°	
5	300·4	50	3474·5
10	603·1	55	3968·0
15	910·5	60	4527·4
20	1225·1	65	5178·8
25	1550·0	70	5966·0
30	1888·4	75	6970·3
35	2244·3	80	8375·3
40	2622·7	85	10764·7
45	3030·0	90	∞

It is clear that this projection can never reach the pole, but it may be repeated round the Equator to such extent as may be found useful.

The special feature of this projection is that a straight line on the map represents a rhumb line on the globe—that is, a line which cuts all the meridians at the same angle. The effect of this in a general way has been described in the first part of this treatise, and further details will be given at the end of this chapter.

There is a curious projection, very little used, called CYLINDRICAL PROJECTION. In this projection (figs. 14 and 40) the various points on the surface of the sphere are projected on to the surface of a circumscribed cylinder at right angles to the axis of the cylinder. The effect is to give a projection of equal areas. The degrees of longitude remain the same throughout, as in Mercator; but while in Mercator the degrees of latitude are increased towards the pole, so that they may continue to

maintain the same proportion as on the sphere, in cylindrical projection they are diminished in the inverse proportion, so that the rectangular figure on the map

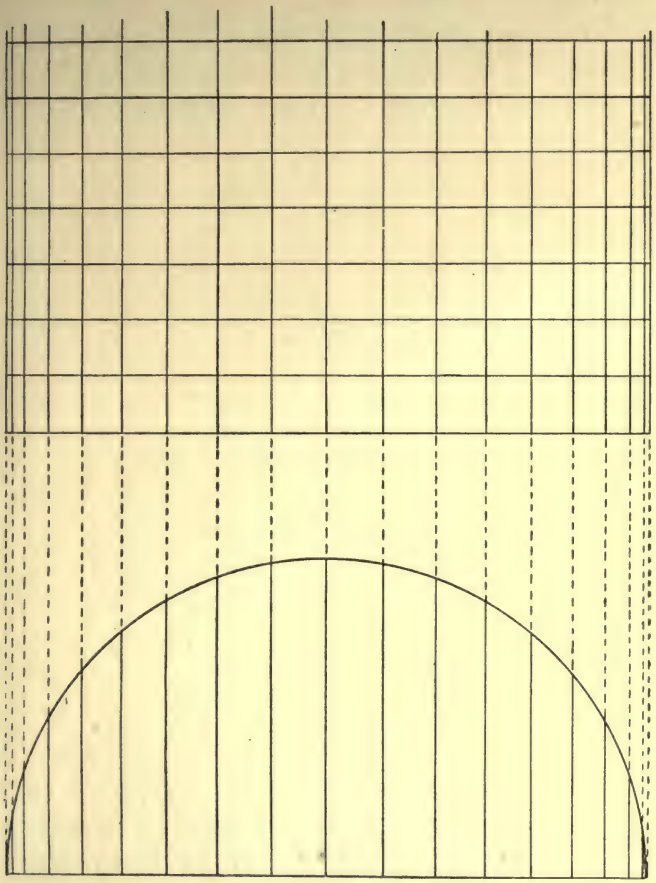


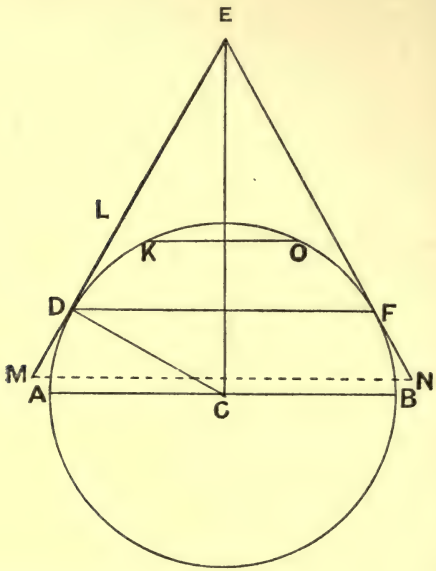
FIG. 40

bounded by parallels and meridians agrees in area with the quadrilateral figure corresponding to it on the globe. This projection is interesting, but not directly very

useful for general purposes, on account of the great distortion near the poles; but it is distinctly useful as a corrective to Mercator, because the distortion is less than on that generally used map.

The next system of projections to be considered is CONICAL, a system which includes an endless number of varieties. It is well known that the surface of a cone is

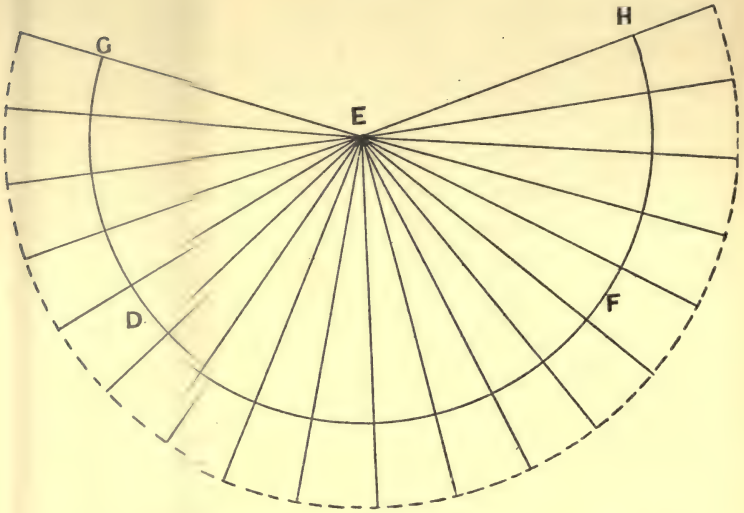
FIG. 41



what is called a developable surface—*i.e.*, it can be laid flat if cut open. In fig. 41 let the circle represent the outline of a globe of which A B is the Equator and the lines E M, E N, the outline of a cone touching the sphere at the parallel D F. Let fig. 42 show the conical surface laid flat, the line G D F H being the line where the surface

of the cone touched the surface of the sphere along the parallel D F. This line G D F H will of course be the development of the parallel D F; and if it be divided into 360 equal parts, and radii drawn through the points of division from E, these lines will be the development of the meridians of longitude. So far there is no difficulty. When, however, it comes to laying

FIG. 42



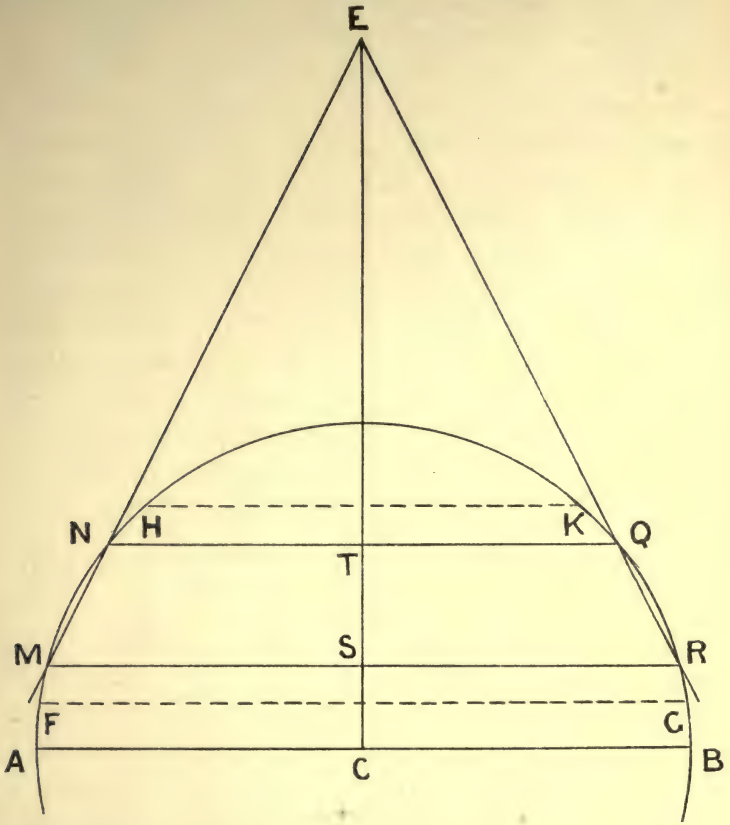
down other parallels, say K O, the question arises how is the point K, fig. 41, to be transferred to the surface of the cone at K? There are three evident ways. First, K may be projected at right angles to the cone. Second, it may be projected in the direction of the radius C K produced; or third, D L may be made equal to D K. The first plan gives an approximation of equal areas, but with distortion, because the length of the degrees of latitude,

which are the same as on the sphere at the line of contact, decrease as they get farther and farther from the line, while a slight consideration of the figure will show that the degrees of longitude, which also are the same length as on the sphere at the line of contact, grow longer than on the sphere as they get farther and farther from that line. The second plan has this advantage, that the degrees of latitude increase along with the degrees of longitude, so that the proportion is very nearly the same as on the sphere, though not exactly so, but the system is little used. The third plan not only has most good points, but is applicable to modifications of the projection to a greater extent than either of the others. It is clear that, if this plan be adopted, the area contained in the quadrilateral composed of any two meridians and any two parallels will be larger on the projections than on the sphere. The projection, however, is good for a moderate area, though correct only along the parallel of contact.

There are several modifications of this projection classed under the name *secant conical.* The simplest of these projections is the following. In fig. 43 let the circle represent the outline of a globe of which A B is the Equator, and let it be required to make a projection showing the band of the earth's surface between the parallels F G and H K. The cone is supposed to cut the surface of the sphere at the parallels N Q and M R, so that its surface lies partly within and partly without the sphere. This problem will be considered, first, in the case of a true sphere, and, secondly, in the case of a spheroid, and here, as well as everywhere else in this

treatise when actual sizes have to be taken into account, the earth will be assumed to be a true sphere whose radius is 20,889,000 ft., or a spheroid whose equatorial

FIG. 43



semi-diameter is 20,923,600 ft., and polar semi-diameter 20,853,657 ft. The figures assumed in the Trigonometrical Survey of India are 20,922,932 ft. and 20,853,375 ft.,

while Clarke gives 20,926,202 ft. and 20,854,895 ft., but those adopted here have been taken by Chauvenet and Loomis, and the table of lengths of degrees given at the end of the treatise are founded on them ; moreover, they are adopted in Chambers's Mathematical Tables, which have been referred to as convenient students' tables.

The matter is of no importance in an elementary treatise, but is mentioned so as to explain certain fractional differences which the student may find if he compares lengths given here with those given elsewhere.

It will probably simplify the matter to the young student if a definite case is taken.

In fig. 43 let the lines F G, H K represent the parallels of 8° and 40°, and the lines M R, N Q those of 16° and 32°. Now the distance of M from the polar axis is clearly R cos lat. M, or R cos 16°, and the distance of N from the same axis is R cos 32°, while the distances C S, C T measured on the axis are R sin 16° and R sin 32°.

Now, if S and T are supposed to be on the polar axis :

$$\begin{aligned}
S M &= R \cos 16^\circ = 20{,}079{,}785 \text{ ft.} \\
T N &= R \cos 32^\circ = 17{,}714{,}875 \text{ „} \\
C S &= R \sin 16^\circ = 5{,}757{,}790 \text{ „} \\
C T &= R \sin 32^\circ = 11{,}069{,}480 \text{ „} \\
S M - T N &= 2{,}364{,}910 \text{ „} \\
S T &= C T - C S = 5{,}311{,}690 \text{ „} \\
M N &= \sqrt{S T^2 + (S M - T N)^2} \\
&= 5{,}814{,}368 \text{ ft.}
\end{aligned}$$

or M N may be calculated as follows :

$$\tan N E T = \frac{S M - T N}{S T}$$

$$\begin{aligned}
M N &= S T \sec N E T \\
&= 5{,}814{,}368 \text{ ft.}
\end{aligned}$$

Now, in this length the side of the cone has approached the axis 2,364,910 ft., and it will consequently meet the axis at a distance from N of M N $\frac{T N}{S M - T N}$ =43,553,800, or a distance from M of 49,368,168 ft.

If, now, in fig. 44 any point, E, be taken, and two circles be described with radii of 43,553,800 ft. and 49,368,168 ft. measured on the scale of the map, these two lines will represent projections of the parallels of 16° and 32° respectively.

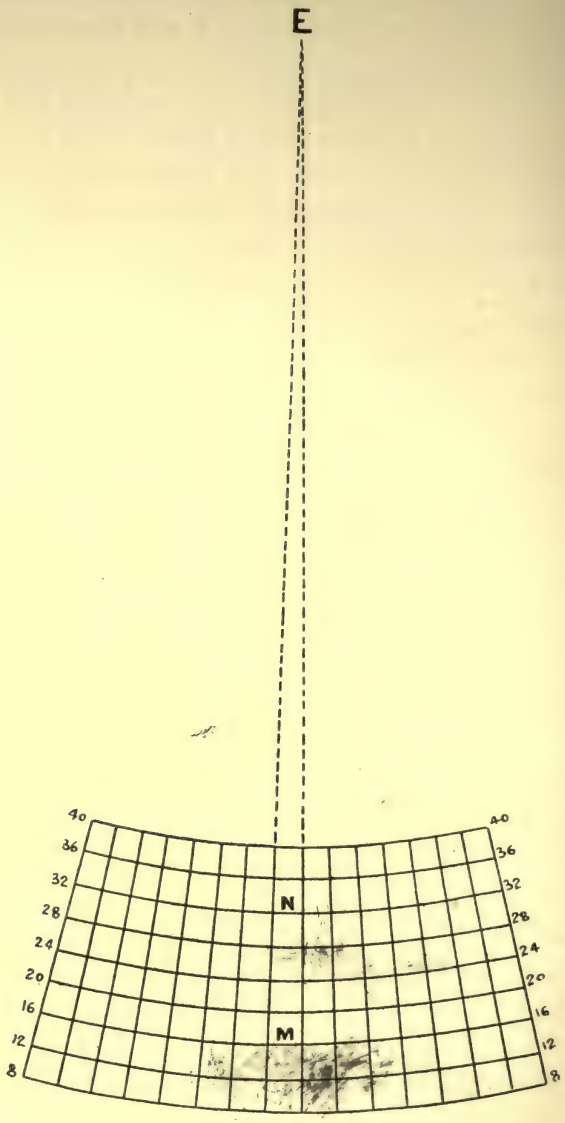
The simplest way of representing the other parallels is to divide M N into a certain number of equal spaces, say four, and describe circles through the points of division with E as centre. These circles will represent the parallels of 20°, 24°, 28°, and by laying off two equal lengths south of M and north of N, the parallels of 8°, 12°, 36° and 40° can be shown.

To find the meridians, find the length of a degree at 16° by dividing the length S M (fig. 43) by the number of degrees in an arc equal to radius, say $\frac{20,079,785}{57{\cdot}2957795}$ = 350,458 ft.

Using this as unit, divide the parallel of 16° into lengths of 1° or 4° or 10°, as required, and draw lines from E through the points of division. In the figure the lines are drawn at intervals of 4°. The usual method of finding the position of the meridians is to calculate the angle contained between two adjacent meridians at E, but the above method is probably clearer to the young student

The above problem is very simple, but the working

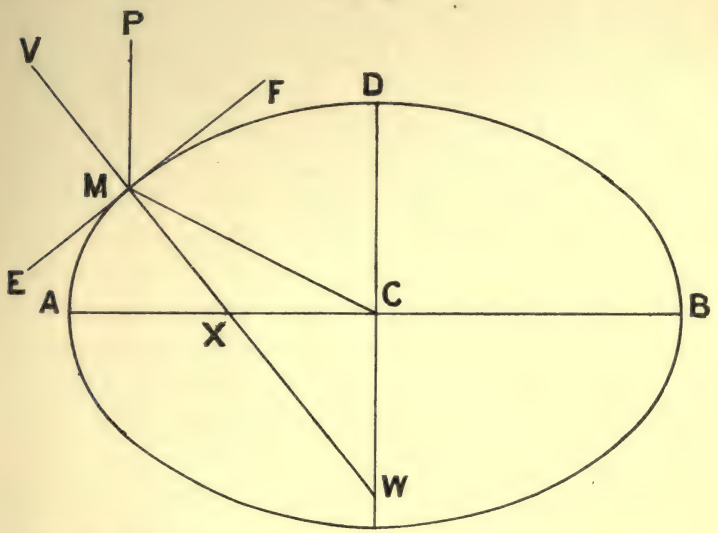
FIG. 44



out is troublesome on account of the large number of digits, but the following problem is rather more difficult.

Instead of a sphere let the earth be assumed to be a spheroid of the dimensions above stated, and here it becomes necessary to define latitude more clearly. In fig. 45 let the ellipse represent a section of the earth

FIG. 45



through the poles, the flattening at the poles being much exaggerated. Let E F be tangent to the ellipse at the point M, or in other words let it be the horizontal plane for an inhabitant at that point. Draw V M X W at right angles to E F, and it will represent the position of the plumb line at M. Draw M P parallel to the polar axis, and as has been explained it will point in the direction of the visible pole of the heavens (the distance being

such that all lines from any part of the earth towards the pole are parallel). Then the geographical latitude of the point M is the angle M X A, or it is the equal angle F M P—*i.e.*, it is the angular altitude of the visible celestial pole. (It is true that in some parts of the world the plumb line is affected by the attraction of mountain ranges, &c., and does not hang precisely in the direction in which it would hang if the earth were a true and uniform spheroid, and consequently there is a minute difference between what is called astronomical latitude, or latitude fixed by observing the heavenly bodies, and geographical latitude fixed by measurement on the globe, a subject which is at present under investigation in India by Major Burrard, R.E.; but the difference in any case is not only very trifling but extremely difficult of detection, and is here disregarded.) This geographical latitude, however, which is the latitude adopted in maps, is entirely different from the angle M C A which is the geocentric latitude of the point M, and this affects all the calculations. The cross-section of the earth through the poles is assumed to be an ellipse, and in any ellipse if the major semi-diameter be a and the minor semi-diameter be b, the eccentricity, e, is $\sqrt{\frac{a^2-b^2}{a^2}}$, and the length of the line M W, which is M V produced, is $\frac{a}{\sqrt{(1-e^2 \sin^2 l)}}$, where l is the geographical latitude of M, and the distance of M from the polar axis is clearly M W sin M W D or M W cos l.

The distance of M from the equatorial axis is

$$(1-e^2)\,(\text{M W} \sin l).$$

Taking the particular case of $l = 16°$ the following results are arrived at:

$$\text{M W} = 20{,}928{,}900 \text{ ft.}$$
$$\text{M W} \cos 16° = 20{,}118{,}150 \text{ ,,}$$
$$(1 - e^2)\ (\text{M W} \sin 16°) = 5{,}730{,}280 \text{ ,,}$$

Referring back to fig. 43, assume the curved line to represent a section of the earth through the poles drawn to scale—*i.e.*, an ellipse which is so nearly a circle that the difference cannot be detected by the eye. It will at once be seen that the above calculations give the lengths S M and C S on the assumption that the earth is a spheroid of the dimensions previously stated.

The lengths C T, T N are calculated in the same manner, and the results are as follows in feet (it being remembered that the points C, S, T, on fig. 43 are on the polar axis):

S M 20,118,150	C S 5,730,280
T N 17,760,870	C T 11,024,150

The length M N is calculated as before, and is

$$\sqrt{(\text{S M} - \text{T N})^2 + \text{S T}^2} = 5{,}794{,}981$$

Then, on the same principle as before,

$$\text{E N} = \text{M N}\ \frac{\text{T N}}{\text{S M} - \text{T N}} = 43{,}662{,}150$$

and

$$\text{E M} = 49{,}457{,}131.$$

These, then, are the radii of the circles representing 16° and 32°, and the other parallels are laid off as in the last example. The meridians are found as before by finding the length of a degree on parallel 16°,

$$\text{say } \frac{20{,}118{,}150}{57{\cdot}2957795} = 351{,}128 \text{ ft.}$$

The appearance of the map on the scale of the diagram in this treatise will be the same as in the last example.

This is the projection adopted for the new large map of India and surrounding countries. It is true that in that case the map extends from lat. 4° to lat. 40°, but the southern strip between 4° and 8° is almost entirely ocean, and the projection has been designed to fit the area between 8° and 40° as above.

An important variation of conical projection much superior to the one just described is as follows:

Take any line, such as the line M N on fig. 44, as the central meridian, and lay off along it the degrees of latitude belonging to the spheroid as given in the table. Thus, take half the length of the degree at 16°, the full length of the degree at 17°, 18°, 19°, and half the length at 20°, and add them together to get the length between 16° and 20°, &c., as follows:

	Feet
16° to 20°	1,452,387
20° „ 24°	1,453,039
24° „ 28°	1,453,793
28° „ 32°	1,454,636
16° „ 32°	5,813,855

Now, by the table at end of treatise it is seen that

1° of longitude at 16° latitude = 351,128 ft.
1° „ 32° „ = 309,986 „
Difference 41,142 „

At this rate of approach two meridians will meet at a distance of 43,804,730 ft. from N, or 49,618,585 ft. from M, which lengths, it will be seen, are appreciably different from those found by adopting the secant conical

projection of the spheroid. Taking these lengths as radii for the parallels of 16° and 32°, the positions of the other parallels are found by laying off (of course, according to scale) the true lengths of the degrees of latitude as given in the table. The meridians are found by laying off the true lengths along parallel 16°, and drawing towards the centre, when the parallel of 32° will have degrees of the true length.

The degrees of longitude on the parallels between 16° and 32°, as before, will be too short on the map, and those on parallels outside of these limits too long on the map.

In general appearance on a small scale this projection will be similar to that on secant conical projection, but on a scale of, say, $\frac{1}{1000000}$, the difference will be very marked.

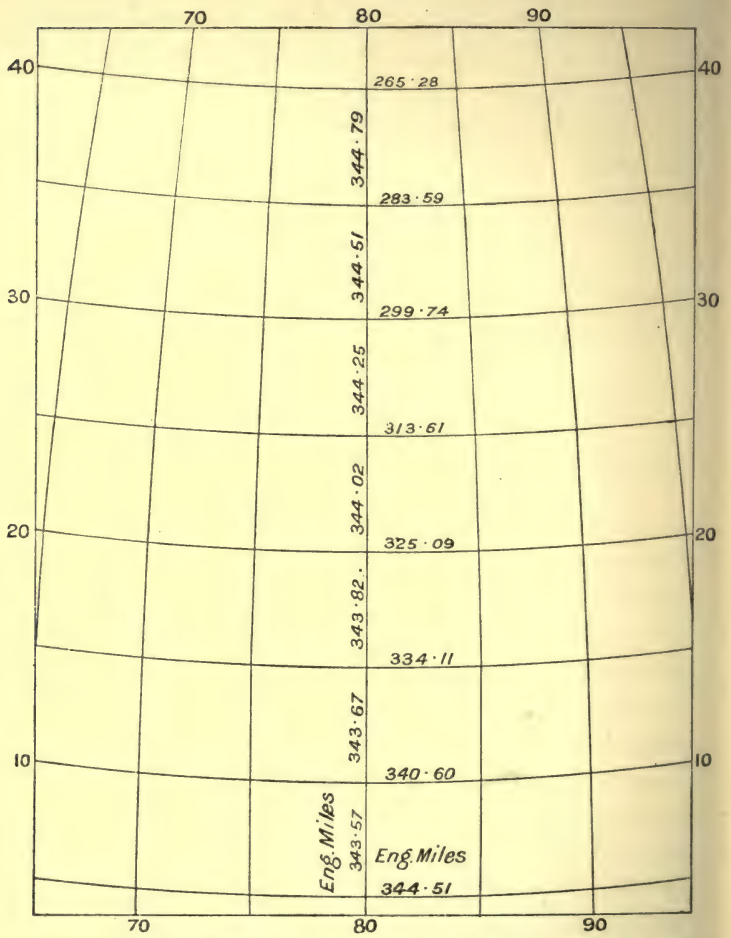
This last described variation is sometimes named 'secant conical,' but the name does not appear to be suitable, and should be confined to the projections previously described.

In all the above described conical projections the meridians and parallels cut each other at right angles.

This is not the case in the very useful modification of conical projection which naturally follows on the modification last described. Assuming that modification to begin with, and choosing two suitable parallels as standard parallels, find the centre and describe the parallel circles as before, but set out degrees of longitude of the same length as on the sphere, not only along the standard parallels, but along every parallel, and draw meridians through them. The result will be as

in fig. 46. In this modification there is very little discrepancy in area, but considerable distortion in shape

FIG. 46

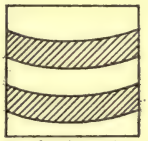


at the corners of the map. In the modification described immediately before it there is very little distortion in

shape or size, but what there is is distributed over a greater area. The diagrams show by shading roughly the good portions of maps where these systems are adopted. The straight meridians may be best for a country like China, and the curved for a country like India.

Conical projection, as has been said, admits of almost endless modifications, and it is unnecessary to describe all, but there is an important one which should be mentioned. When a cone whose apex is in the prolongation of the axis of the sphere touches the sphere, the line of contact is a parallel of latitude, and on the development of

Curved Meridians

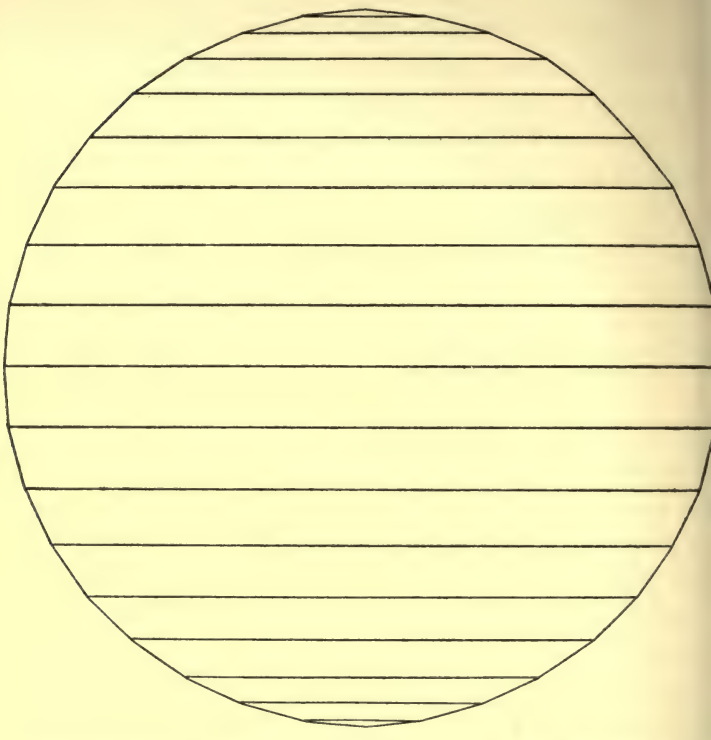


Straight Meridians

the surface of the cone it becomes a circle whose radius is tangent of co-latitude (or cotan latitude), the radius of the sphere being unity. Now, suppose the sphere to have its surface divided into a number of conical surfaces as in fig. 47, and further let the number be so great that the difference between the figure and a sphere is inappreciable, then each strip of the surface can be developed as a narrow band of equal width throughout forming part of a circle, the radius of the centre line of the band being cotan latitude, and if all the bands be laid out flat they will appear as in fig. 48. This can be used as the basis of a very good projection, thus: Take any line (see fig. 49) to represent the central

meridian of the map; assume a radius for the sphere and mark off equal lengths on the meridian, each equal to a degree corresponding to that radius; then through each point of division describe a circle having its centre

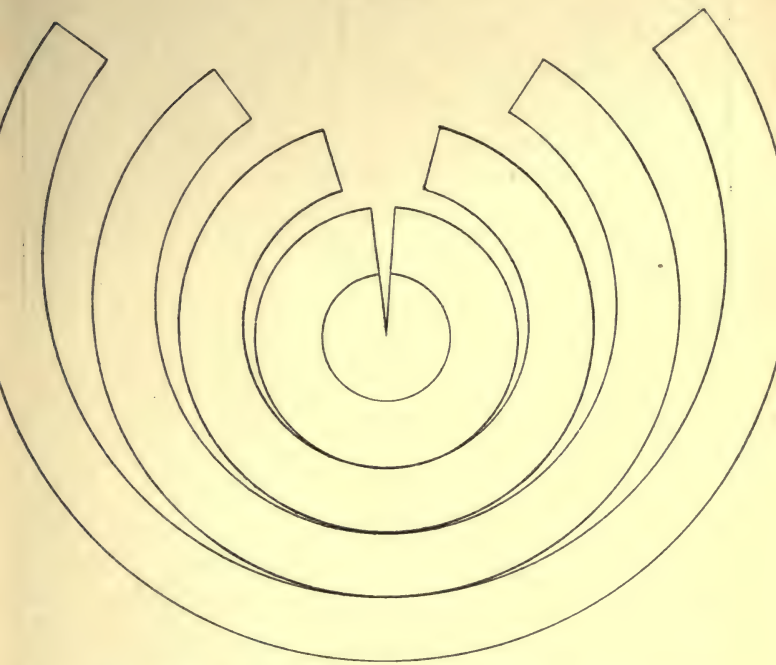
FIG. 47



in the central meridian and having a radius equal to cotan latitude to the radius assumed. Mark off degrees of longitude on each parallel (in practice every fifth or tenth may be enough) and draw curved lines to represent meridians. This suits well for a map with great

difference of latitude and comparatively little difference of longitude, and can be used advantageously in areas crossed by the Equator, such as Africa. In this respect it differs essentially from the simple conical projection, which is suitable only for moderate differences of latitude, but can take in great differences of longitude.

FIG. 48

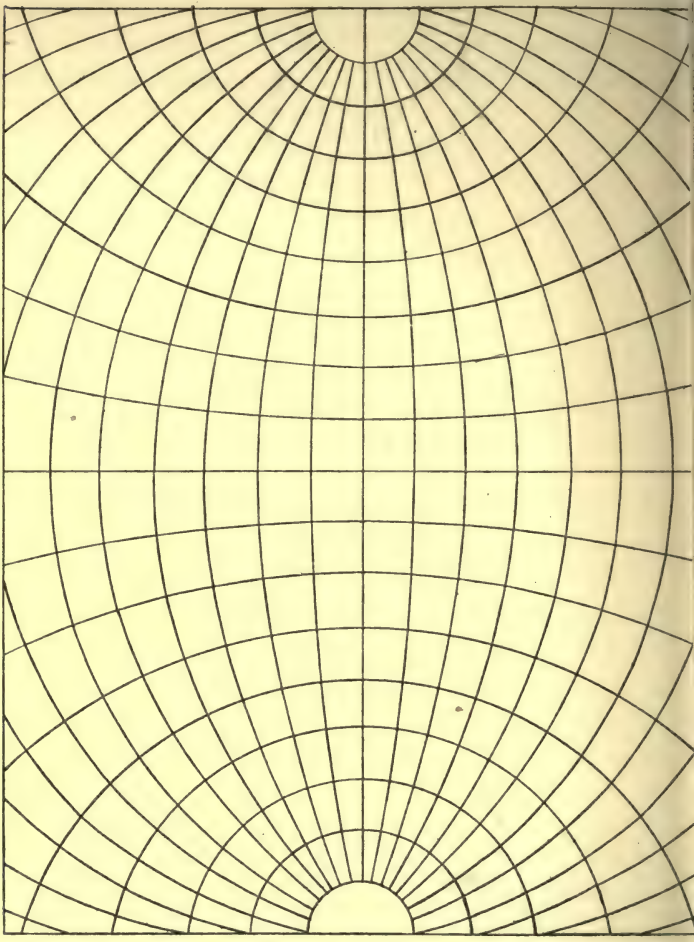


There are other methods of drawing the meridians of longitude on this projection besides the one above described, but the one given appears on the whole to be the most satisfactory.

There are several other varieties of conical projection,

some applicable to areas including great differences o
both latitude and longitude, such as Asia, or Europe an

FIG. 49



Asia; and others where by the introduction of some
refinement a balance of errors, or a least maximum

error, or some other special quality, is obtained, and where, as likely as not, the shape of an important coast line is sacrificed in order to avoid distortion in some quadrilateral enclosing nothing but ocean. The latter series does not seem to merit much attention, but the former—viz., the series of varieties suitable for large areas—is important. Their consideration, however, may be left to those students who take up the matter seriously and proceed to study the subject from the point where this treatise ends, or rather perhaps, having gained some general ideas from this treatise, begin to study the entire subject *de novo*.

The number of projections now described, however, is sufficient to form a basis on which to found a general consideration of those projections used for showing the whole or a large portion of the earth's surface. There are, in the first place, those projections in which some one definite property of the surface of the sphere has been mathematically preserved and is directly made use of. Mercator and gnomonic projections are good examples. Gnomonic projection, when used, is used solely for laying down great circle routes, and if it is not used as much as it might be at sea, this is to some extent to be accounted for by the fact that the best vessels run on regular routes, and their course once laid down holds good for long periods. In Mercator's projection the fact that a rhumb line is shown by a straight line on the chart is its essential property, and this property is made use of at sea not merely from day to day, but from hour to hour. Maps or charts on those projections are perfect for the purposes for which they are employed, and no

deviation from the strict principle of construction can be permitted.

For the purpose of giving general ideas of the world, or large portions of the world, however, they are distinctly bad. Gnomonic projection is probably the worst imaginable, but luckily is rarely used for this purpose. Mercator, however, is a good second, and, unfortunately, it is used almost universally as a map for general or commercial purposes, and more than that, it is used to a considerable extent in the teaching of elementary geography. For this latter purpose there is absolutely no excuse, and it ought to be abolished from all school atlases. Provided people have a clear conception of the relative shapes, sizes, and positions of the various continents, the use of a Mercator map as a diagram for commercial purposes may be allowable.

In stereographic projection all lines cut each other at the same angle as on the globe, and all circles on the globe appear as circles on the projection. These properties are made use of in making the projection and the projection when made is useful for working out certain questions—such, for instance, as transforming right ascension and declination into altitude and azimuth in astronomical problems, and in the hands of experts it is useful as an intermediate step in constructing other projections, and also for other special purposes; but the fact that small areas in any part of the sphere are projected into similar shaped areas in the projection is the quality generally mentioned in connection with the projection, and spoken of as if it were a useful quality,

and one that entitled the projection to take high rank as one for giving good general ideas. For this in the author's opinion there is no excuse. One does not consult a map of the world to see the shape of Ceylon, but to see the shapes of continents, and the distortion of Africa for instance is much greater than in many other projections. As regards shape, then, stereographic projection is misleading in the case of all continents of considerable size, while in the case of small places, like Ceylon and Iceland, the shape of each is well preserved, but the latter is shown four times as large as it would be if plotted on the same scale as the former. The high reputation that some projections have enjoyed appears to have arisen from map projection having been treated too much as a branch of mathematics. The absolute mathematical accuracy with which special properties of the sphere are retained in gnomonic and Mercator projection cannot fail to excite admiration, while the many beautiful and interesting relations between the sphere and its representation in stereographic projection which exist and seem to multiply on a close investigation have an irresistible fascination for anyone interested in the subject, but against all this the map maker should be on his guard.

For giving good general ideas of the whole world probably elliptical equal area projection is as good as any, while for a hemisphere, zenithal equal area has many good points. Both of them are strictly mathematical projections. Whether any one or other of the forms of globular projection has any advantage over an arbitrary projection of approximately equal zenithal

distances, but having parallels and meridians consisting of straight lines and circles only, is a question on which authorities may hold opposite opinions, but the author is inclined to believe that there is not a scrap of difference between them. When it comes to areas less than a hemisphere, no general statements can be made. In the case of large areas, like Asia or Africa, the decision as to the best projection to be adopted has to be arrived at on the same basis as in the case of a hemisphere. If, for instance, long railway routes through Europe or Asia have to be considered, the map on gnomonic projection (fig. 17) should certainly be consulted, but for giving general ideas and teaching elementary geography some form of conical projection is preferable. In the case, however, of maps of moderate areas an entirely new feature is introduced. For such areas the absolutely ineradicable distortion may not be so great as to interfere with measurements, and then even for general purposes the details of projection become important. Shape, area, accuracy of linear dimensions in one or more directions are all important, and even the best authorities may differ as to which should be sacrificed in any particular case; but this much may be asserted, that, having adopted any projection which is not strictly mathematical, such as one of the conical series, the same standard of accuracy should be adhered to throughout. To adopt, say, the secant conical, to assume a spheroidal form for the earth with semi-diameters stated to a foot, to calculate the length of the side of the cone contained between the two cutting lines also to a foot, and then to divide this into as many equal parts as there are degrees on the spheroid

between them, and adopt the length of a degree of latitude so found throughout the map, in spite of the fact that a degree at the north of the map differs from one at the south by some 1,500 ft., and that the length chosen is more than 500 ft. shorter than the *shortest* degree on the earth, does not seem reasonable, and looks as if the mere computer had been able to incorporate his own somewhat contracted ideas into a generally good scheme, and to turn out a mere mathematical curiosity under the impression that map projection is simply a branch of mathematics instead of an art on a mathematical basis.

Since the issue of the first edition of this treatise, a very useful pamphlet has been compiled by Major Close, giving what is practically a complete list of all map projections worthy of notice, with a short description of their special properties, the whole forming a valuable index to the literature of the subject.

He makes four good suggestions, the first, third, and fourth of which are :

(1) That there should be a general agreement as to the names to be used, and that the issue of an authorised list by the Royal Geographical Society would do much to settle this.

(3) That by a general agreement of geographical bodies and map makers, the name of the projection used should be printed on all maps.

(4) That on all equal-area maps a scale of areas should be given (say 1 sq. inch = 100 sq. miles).

The fourth suggestion can and ought to be adopted by each individual map maker at once.

The third suggestion hangs somewhat on the first because of the uncertainty as regards the projections indicated by the various names ; but if names can be defined, the suggestion should be adopted by those who approve, and others would soon follow.

The first suggestion requires definite action by some leading body, and practically the Royal Geographical Society is the only one that can act. That society might pick out, without reference to classification, a certain number, and a reasonable number only, of useful projections, defining each distinctly, and assigning to each a name and a number, arranging them in some arbitrary manner, alphabetically or otherwise.

These might then be arranged in classes, such as Perspective, Zenithal, Equal Area, Conical, &c., and the fact that a projection appeared in two or three classes would cause no confusion, but seeing that the classes are not mutually exclusive, the introduction of an attempt at classification in the original list lands one in hopeless difficulty.

Major Close's third suggestion is in favour of the publication of an English edition of 'Germain,' or an English text-book founded on 'Germain,' and Colonel Clarke's article in the 'Encyclopædia Britannica.' The exigencies of space have necessitated the curtailment of the last-named article as regards the treatment of gnomonic and some other projections, and an expansion of it in book-form would no doubt be valuable, but so far as it goes it is hard to see how it could be improved upon. Germain is, of course, a standard work, and being out of print is not readily accessible but the

trouble and expense attendant on a re-issue without the slightest chance of any return stand in the way of the work, as public bodies may well think that, so long as the book is available at all, it is sufficient for deep students of the subject.

CHAPTER IV

PROJECTION OF SMALL AREAS

HAVING given a general description of some of the principal projections applicable to maps of the world, or of large portions of the world, and of the mathematical basis of these projections, it may be well to touch briefly on the question of the projection of small areas such as single countries, or even single counties.

The maps to which reference is here made are such maps as those of the Ordnance Survey of Great Britain. This Survey is published on several different scales. Town plans are issued on a large scale, sometimes as large as 10 ft. to the mile (though these are no longer made at national expense), but the whole country has for years been published on scales of 25·344 ins. to the mile, 6 ins. to the mile, and 1 in. to the mile, and quite lately on a scale of 4 miles to the inch. The town maps may be considered as surveys of plane surfaces, and therefore need not be further referred to. The 6-in. maps and 25-in. maps are on the same projection, a sheet of the 6-in. map being merely a reduction of a certain number of sheets of the 25-in. map, each of the last named of which shows an area

measuring $1\frac{1}{2}$ mile in length and 1 mile in width. The large sheets of the 6-in. maps, which cover as large an area as 16 sheets of the 25-in. map, are of old date and are going out of use, while the smaller sheets which are up to date, and which cover the same area as 4 of the 25-in. sheets – *i.e.*, an area measuring 3 miles by 2 miles – are gradually becoming the only ones kept in stock.

Each sheet of the new edition of the 1-in. map shows an area measuring 18 miles by 12 miles, and 25 of these go to make 1 sheet of the 4 miles to an inch map, which is on the same projection, and which shows an area of 90 miles by 60 miles.

As the projection of the 1-in. map differs somewhat from that of the 6 in. map, it will be necessary in treating of the British Ordnance Survey to consider both of these projections.

It here becomes necessary to give a brief description of the method of making large surveys, but the reader must clearly understand that the object of this treatise is not to enable him to make surveys, but simply to understand the main principles on which they are based.

All surveying is based on measurement, and, in fact, the only way to find out the size of the earth is to take a chain or a rod of known length and measure it. It so happens, however, that of all operations in the field direct measurement is the most difficult, while measuring angles with a theodolite is one of the easiest—that is to say, if three capable surveyors are told to measure a long line with a chain, unless great precautions are taken they will give results differing from each other by appreciable amounts, while if they are told to measure

certain angles, with ordinary care the results will closely agree. In practice, therefore, one line is measured several times with the greatest care and is extended by triangulation.

Let fig. 50 be a diagram of triangles *on a plane*; let A B be a measured line 20,000 ft. long, inclined 5° to the west of north; let the various angles marked be those observed with a theodolite: then by solving the triangles the position of all the points marked can be calculated with reference to A, a few results being given in the following table. In practice, it need hardly be said, the triangulation is much more complex and numerous checks are introduced.

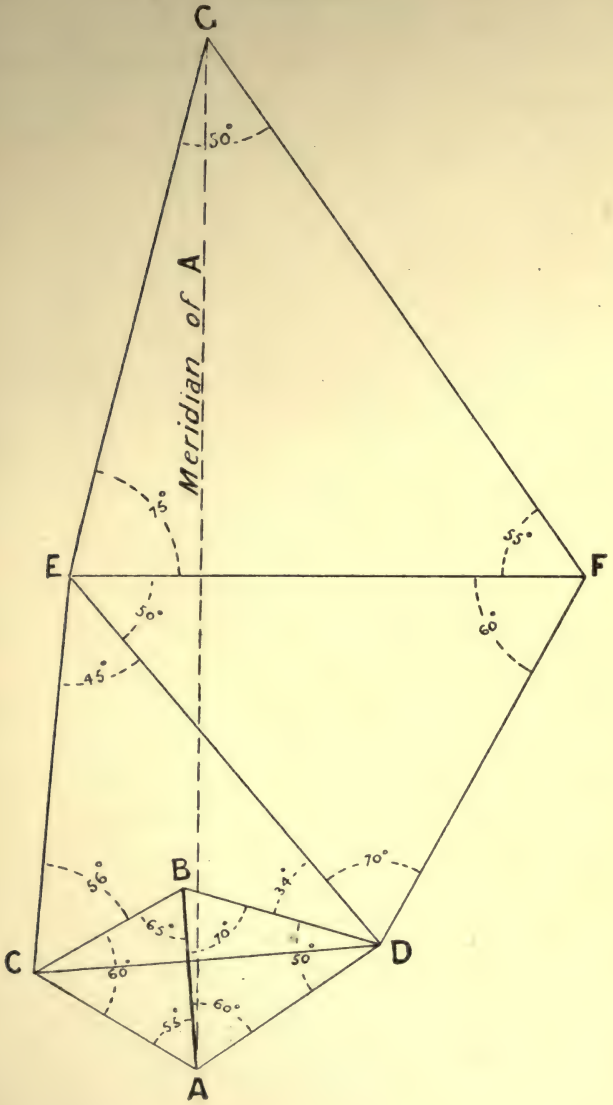
Lengths	Bearings	Northing	Easting
AB 20,000	5 W. of N.	+ 19,924	− 1,743
BC 18,917			
BD 22,610	15 S. of E.	− 5,852	+ 21,840
CD 38,391			
DE 53,565	41 W. of N.	+ 40,427	− 35,142
EF 58,121			
EG 62,151	14 E. of N.	+ 60,305	+ 15,036
AG		+ 114,804	− 9

Co-ordinates of main points

	Northing	Easting
A	0	0
B	+ 19,924	− 1,743
	− 5,852	+ 21,840
D	+ 14,072	+ 20,097
	+ 40,427	− 35,142
E	+ 54,499	− 15,045
	+ 60,305	+ 15,036
G	+ 114,804	− 9

Now, if a similar set of triangles be set out on a sphere, the first thing that will be noticed will be that the

FIG 50



sum of the three angles of each triangle will be greater than two right angles. If, however, $\frac{1}{3}$rd of the excess be deducted from each angle, the triangle can be dealt with as a plane triangle with extremely accurate results, and the angles given on the diagram may be assumed to be the angles after the correction for spherical excess has been applied. When the system does not extend far to one side or the other of the meridian, as in the figure, the final result—viz., that G is 114,804 ft. north of A and 9 feet at right angles to the west of it—will be the correct result measured on the sphere. Now suppose the latitude of A be carefully fixed, and the latitude of G be also carefully fixed by astronomical observation, and suppose there be no local disturbing causes, and suppose the difference of latitude be found to be 0° 18′ 48″, then a degree would measure 364,793 ft., and if the earth be a sphere, the radius of the sphere would be about 20,901,100 ft. Naturally, much greater lengths are taken in practice, but this is an outline of the principle.

Now, measurements of degrees at various places on the earth have shown that they differ in length at different latitudes, and that consequently the earth is not a sphere but a spheroid, and the radius found, as above, is not the radius of a sphere, but the radius of curvature of the meridian at the latitude in question. If, then, a degree could be measured at each latitude, and the radius of curvature found, and all the 90 little pieces each representing a degree at each degree of latitude, from Equator to pole, could be joined together, they would give the shape of the earth. Each degree has not been measured,

but a large number have, and taking all these into account, the cross section of the earth through the poles has been found to be as nearly as possible an ellipse, the semi-equatorial diameter of which is 20,923,600 ft., and the semi-polar diameter 20,853,657 ft.

This result having been arrived at, no further reference is made to actual measurements of arcs which have been carried out, but each degree of the meridian has been re-calculated on this basis, as also a degree of longitude at each degree of latitude, and the results are given in the table at the end of the treatise, which is condensed from a table given in Loomis's 'Astronomy,' published in 1870. The following approximate formulæ may interest the student:

Length of a degree of latitude in feet at latitude ϕ
$$= 364{,}570 - 1{,}820 \cos 2\phi.$$
Length of a degree of longitude in feet at latitude ϕ
$$= 365{,}492 \cos \phi - 307 \cos 3\phi.$$

In working out any system of triangulation on the earth, all the triangles are calculated on the assumption that they are on the surface of a spheroid of the dimensions given, and the position of each point forming the apex of a triangle is eventually calculated with reference to meridians and parallels—*i.e.*, its latitude and longitude are fixed with extreme accuracy. No doubt the latitude and longitude could be fixed by astronomical observation, but this would entail building a regular observatory at each point and observing for months, while in practice extremely careful observations are made at one or two observatories only.

The sides of the main triangles of a survey may

average some 30 miles in length, though larger sights are often taken. After the main triangulation is finished, each triangle is cut up into smaller triangles where, as the errors do not accumulate, so much accuracy is not required. After that, the plan is filled in by ordinary surveying by theodolite and chain, or by plain table or otherwise, but always on the assumption that the surface surveyed is a plane, spherical distortion being inappreciable.

As far as map projection, therefore, is concerned, the work is confined to plotting the corners of the various triangles on paper. They can, of course, be plotted on a sphere at once, but on a plane piece of paper difficulties arise, and though in a sense they are of the same sort as those that arise in making maps of the earth, the amounts which have to be considered are so trifling in the one case compared with the other that the difficulties may be regarded as of a different order.

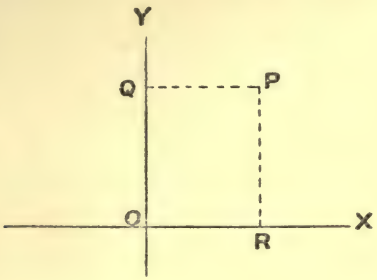
In dealing with fig. 50 it has been stated that the positions of the various points may be fixed by the numbers given in the table ; that is to say, distances along the meridian of A are in fact distances north and south, and distances right and left at right angles to that line may be considered to be distances east and west, of that line. A warning was, however, given that this latter statement was only true if the distances were small, and the table was only actually used to determine the length of the meridian from A to the latitude of G.

Maps have, however, been plotted on this system, which has been carefully worked out by 'Cassini,' and is known by his name or by the name of the system of

RECTANGULAR CO-ORDINATES, and may be described as follows:

On any plane surface, two lines O X and O Y called the axes of co-ordinates (see fig. 51) may be drawn, and the position of any point P may be defined by giving the length O Q measured along O Y and the length Q P measured perpendicular to O Y, and these lengths are the rectangular co-ordinates of the point P. If P R be drawn perpendicular to O X, then the lengths O R and R P will be equal to Q P and O Q, and the position of the point P may be defined by either system

FIG. 51

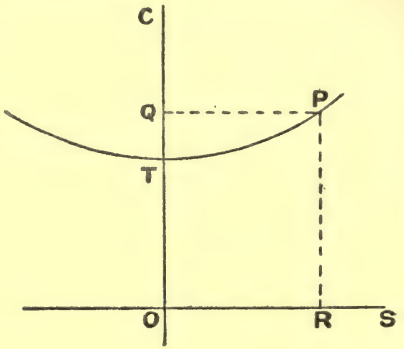


of lines indifferently A system of mapping on this basis is employed for the Revenue maps of India, and is officially described as being based on rectangular co-ordinates computed on the assumption that the earth is a plane. This description, however, is not satisfactory.

The system adopted in actual practice may be described as follows. The meridian passing through the centre of the area to be mapped, such as O C (fig. 52), is chosen as the axis of co-ordinates in one direction. On this some point O, of which the latitude and longitude are known, is chosen as the origin of measurements.

The position of any other point P of which the latitude and longitude are known is fixed by drawing a great circle P Q through P to cut the standard meridian at right angles, and then calculating the lengths P Q and O Q. In practice this is usually done by means of tables from which the length T Q can be taken, T being the point where the parallel of latitude through P cuts the standard meridian. In low latitudes and for moderate distances, the length Q P may be taken as equal to T P, and in any case the difference is easily calculated; and hence the

FIG. 52



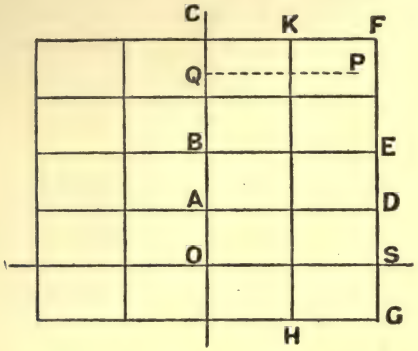
lengths O Q and Q P, which are the rectangular co-ordinates of P, are easily found.

A diagram of sheets is shown in fig. 53, where the line O C is the standard meridian, the lines O S, A D, B E, &c., are the top and bottom lines of sheets, and are great circles cutting the standard meridian at right angles, while the lines O C, H K, G F are end lines of sheets, O C being the standard meridian, and therefore a great circle and the lines H K, G F being small circles parallel to the standard meridian.

The point P and any number of points defined in the same way can then be plotted at once in their proper positions on their own sheets.

So far apparently the same plan has been adopted as on a plane surface, but a little consideration will show that this is not the case. On a plane surface when the position of a point has to be fixed, all that is necessary is to draw a line from that point at right angles to one axis, to measure the length of that line and also the

FIG. 53



distance of the point where it cuts the axis from the origin, or point of crossing, of the two axes. It makes not the slightest difference which axis is taken. In the projection of the earth's surface it may truly be said it makes all the difference in the world. If in fig. 52, O S be a great circle cutting the meridian at right angles at O, and if P R be a great circle through P cutting the great circle O S at right angles in R, it will be found that the lengths P R, O R, are not equal to the lengths

O Q, Q P, consequently points cannot be plotted with reference to either axis, but all points must be plotted from co-ordinates of the class represented by O Q, Q P.

To take an extreme case: suppose O Q and Q P to be each 1,000 miles, then O R and R P on the earth will be respectively about 1,033 and 968 miles, while on the map they will measure exactly the same as the lines O Q, Q P. The earth, that is to say, absolutely refuses to be treated as a plane, and the system of projection breaks down when applied to large areas. For small areas, however, the discrepancies between the two systems of co-ordinates are small, and the projection may be employed so long as it does not exceed certain limits.

For an elementary treatise of this sort the best way of arriving at the practical limits of this projection is to select an easy case.

While in actual practice the standard great circle adopted is invariably a meridian, any great circle can be taken. If the Equator be chosen as the standard, then the subsidiary great circles are as a matter of fact meridians, and the entire system of projection will then be based on a system of great circles crossing at right angles, with which everyone is so familiar that a mathematical view of the question becomes quite simple. The only difference from the usual system is that the standard great circle runs east and west instead of north and south. On the principles stated above, if the intersection of the Greenwich meridian be taken as the origin, then the position of any other point is found as follows. The great circle passing through this point, and cutting the Equator or standard great circle at right

angles, is the meridian passing through the point. The rectangular co-ordinate in one direction is therefore the latitude of the point, and in the other direction it is the longitude *measured on the Equator.* The diagram of sheets will consist of a standard straight line to represent the Equator ; lines at right angles to it, which will represent meridians ; and lines parallel to it, which will be parallels of latitude ; and the map of the entire globe will be as shown on fig. 14A.

All parallels and meridians will cross at right angles, and the only distortion is that arising from the fact that parallels of latitude on the earth decrease in size as they go towards the pole, or, in other words, that the subsidiary great circles converge. All lengths, therefore, measured north and south will be correct, while all distances east and west except on the Equator will be too long on the map. Those, for instance, measured east and west at the parallel of 2° north or south will be shown too long on the map in the proportion of 0·9994 to 1. As this corresponds to a length of $\frac{1}{40}$ of an inch, in a sheet of 40 ins. it is inappreciable for mapping purposes.

Now, if a belt of the earth 2° on each side of the standard great circle, or 240 geographical miles wide, can be shown without appreciable distortion on this system at the Equator, it can be shown along any other great circle, and a very good map of England can be plotted on this system, and as a matter of fact the new edition of the 1-in. map of England and Wales is plotted on one central meridian, consequently all the sheets of this map fit together by their bounding lines (which was not the case with the old edition).

The system with all its advantages has two objectionable features. One has already been mentioned—viz., that at the sides of the map distances parallel to the standard great circle (*i.e.*, distances measured north and south on the sides of England) are too long, and while on any map it is necessary to give and take a little, it seems at first sight rather hard that anyone wishing to buy a map of Kent must buy an incorrect one in order that Worcestershire may be all right. The other is that the bounding lines of the sheets near the sides of the map no longer correspond with the cardinal points of the compass. The top and bottom lines, as has been shown, are great circles at right angles to the central meridian. Now, a great circle running east and west through London will cross the Caspian Sea, pass near Madras, cut Western Australia, pass a short distance south of New Zealand, cut the isthmus of Panama, and reach London again through the West Indies, so that, while it runs east and west at London it lies at an angle of about 52° to the Equator where it crosses it, and even in the case of England a great circle running east and west in the middle of the country is inclined some 3° to the parallel at the east and west boundaries of the map.

Still, as has been shown, the error in any one sheet is very small, and if the system is applicable to any one place in the world, it is to a country like England, surrounded by sea except at the narrow neck where it joins Scotland. If, then, the projection had been adhered to throughout—that is to say, if each sheet of the 1-in. map had contained a definite number of

sheets of each of the larger scales, so that the smaller scale would have been an index to the larger scales—it would have shown at least that those responsible for the work had the courage of their opinions, and all the advantages of the system would have ensued. The rectangular co-ordinate of each point would have been applicable to all the scales, and sheets of the same scale would have fitted in any part of the country. This, however, has not been done.

While the system of plotting by rectangular co-ordinates has been retained in the 6-in. maps of the Ordnance Survey, each county or group of small counties has been plotted on its own central meridian. By this means the objectionable features of the projection have been avoided, but it has already been shown that the first—viz., error in scale—is so trifling, even when the system is applied to all England and Wales, that its abolition is hardly worth noticing, while the second objection—viz., that the boundary lines of the outer sheets would not run north and south, and east and west by about 3°—would cause no inconvenience, and judging from the very fine appearance of the new 4-in. maps could hardly be called unsightly. Two trifling objections have therefore been met at the expense apparently of throwing over all the advantages of the projection. The advantage of having rectangular maps that would join up anywhere is very distinct, but by the system adopted, the maps of adjacent counties are arranged as shown on fig. 54 (where the angle is exaggerated). The nuisance of being unable to join up maps crossed by a county boundary is so great that where a boundary

occurs in the middle of a sheet, the sheet can generally be bought with so much of the adjacent county as is

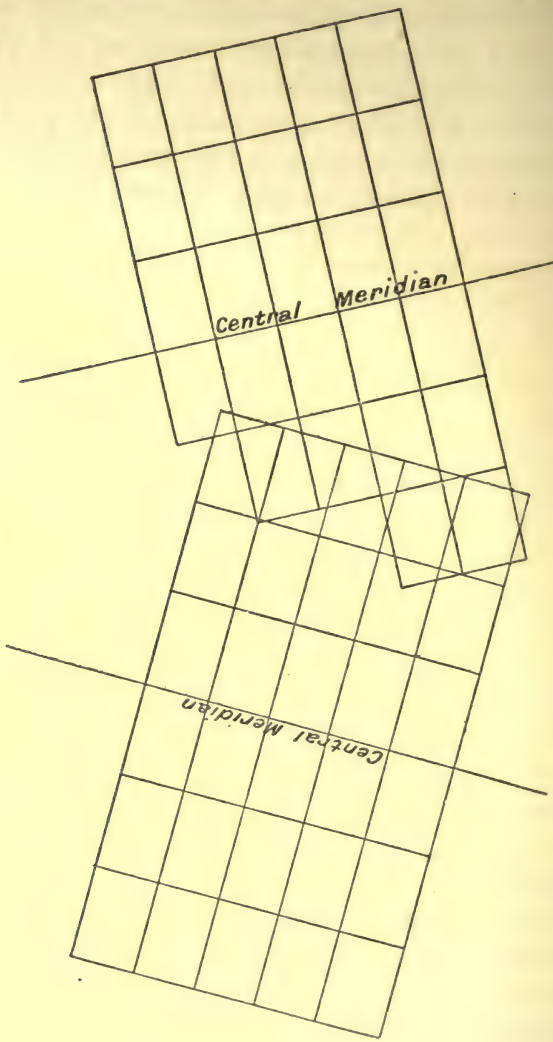


FIG. 54

required to fill it up plotted on the meridian of the county to which the sheet properly belongs. This is a slight palliation in some cases, but does not cover the case of a map being wanted showing considerable portions of adjacent counties. The maps themselves, it must be noticed, where they overlap are so nearly alike that it is difficult to discover differences, but the angles between the bounding lines are so great as to be most unsightly, and thus the very objection which is supposed to be obviated is reintroduced in a most aggravated and aggravating form.

The basis of the county maps having been settled long ago, it is no doubt difficult to change it, but one cannot help thinking that the system bears the impress of a computer accustomed to working out results to a fraction of a foot and appalled by the differences in the co-ordinates of points calculated with reference to different meridians, being unable to grasp the fact that when the final results are plotted to a scale of nearly 1,000 ft. to an inch, errors of several feet are inappreciable.

It is the opinion of the author that the system of plotting by rectangular co-ordinates is altogether unsuitable to the case of selected portions of large areas where eventually one knows the maps must be extended or must be joined to other maps ; that in the case of a country like England it has good points if boldly carried out ; but that carried out in a half-hearted manner, as has been done, it has absolutely nothing to commend it.

It is of course possible to plot a map on any projection and cut it up into sheets of rectangular shape. If

the entire map of England were plotted on conical projection it could be cut up into rectangular sheets, the top and bottom lines of which might be square to the central meridian, or alternatively in the case of the larger scales each county might be divided up into rectangular sheets, the top and bottom lines of which might be at right angles to the central meridian of the county, in which latter case the sheets of adjoining counties would meet as in fig. 54, the lines being inclined to each other at the same angle as the convergence of the meridians, but the overlapping portions of the map would be exactly similar no matter how great the overlap was. The want of similarity in the overlapping portions of maps on rectangular projection arises from the fact that the top and bottom lines, though straight on the map, represent great circles which on the globe do not cut each other at the angle of the convergence of the meridians, and which not being straight constantly change their mutual inclination, so that if the central meridians are far apart and the overlap considerable, the maps may differ to a very great extent.

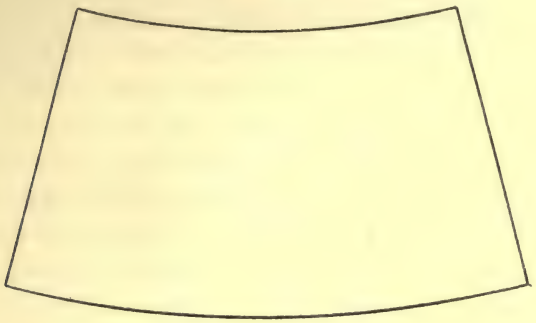
The system of cutting up a map into rectangular sheets for convenience of size must therefore not be confounded with the system of plotting by rectangular co-ordinates.

Another system in very general use is that of plotting the survey on sheets bounded by meridians and parallels.

In great part of the Indian survey the sheets are so bounded, the quadrilaterals so formed being called graticules, and are consequently of the shape shown in

fig. 55. The bounding lines are generally laid down by a system of diagonals, on account of the difficulty of laying down circles of very great radius, and the top and bottom lines are often composed of two straight lines each, the divergence from the circle being inappreciable, but in reality the centre line of each sheet north and south is its central meridian, and the centre line east and west is a circle whose radius is cotan latitude (subject to a slight correction on account of the spheroidal form of the earth), the top and bottom lines

FIG. 55



being circles described from the same centre, and the end lines being the end meridians and being straight.

The advantage of this system is that it is universal. A certain number of sheets can be joined together, say four, or sixteen, or sixty-four, according to the scale, and this applies to *any part of the map*, even if it be a map of Europe, or, for that part of it, of the world, while in the other system sheets plotted on the same meridian (and necessarily within a limited area) can be joined

perfectly, and sheets plotted on different meridians not at all. Further, the latitude and longitude of any point being known, it can be plotted at once on its sheet irrespective of scale, or of the system to which it belongs, and the small-scale map forms a perfect index map to all the other issues.

The disadvantages are that the maps do not join *exactly*, even for an area such as that of an English county, that the shape is not pleasing, and that they are not all of the same size. These latter objections, however, are trifling. In the opinion of the author the balance of advantages is distinctly in favour of the sheets bounded by meridians and parallels.

Among the questions to be decided in connection with this system, one of the most important is the maximum area that can be shown on one sheet without appreciable distortion. This question can be discussed with reference to a sphere because the spheroidal form of the earth will not perceptibly affect the answer. It is true that by the aid of the table of lengths of degrees, given at the end of this treatise, the solution on the spheroidal basis is easy, but in principle the solution on the spherical basis is simplest.

Consider a sheet situated about latitude 45°. In this case it is necessary to find the lengths of degrees of longitude in that neighbourhood, and in a sphere the length of a degree of longitude at any latitude is, of course, equal to the length of a degree at the Equator multiplied by the cosine of the latitude. Assuming a sphere of radius 20,889,000 ft., the lengths of degrees will be as follows :

Latitude	Length of a degree of Longitude
40°	279,280 ft.
42½°	268,794 ,,
45°	257,797 ,,
47½°	246,304 ,,
50°	234,345 ,,

Now, it may be assumed that the point at which trouble commences is the point at which it becomes impossible to lay down the outline of the sheet with straight ends to represent meridians without undue distortion.

Tentatively assume a sheet embracing 10° of latitude with straight ends. Taking the figures from the table for 40° and 50°, the top and bottom lines would measure 234,345 ft. and 279,280 ft. respectively, and the centre line would be the mean of them, or 256,812 ft., while the length on the sphere would be 257,797 ft., a difference of 985 ft., or 0·38 per cent. It is true this difference may be reduced by making the top and bottom lines 0·19 per cent. too long, but even then the difference is too great to be admissible. Again, assume a sheet embracing 5° of latitude from 42½° to 47½°. The top and bottom lines will now be 246,304 ft. and 268,794 ft., making the centre line 257,549 ft., which differs from the length on the sphere by 248 ft., or 0·097 per cent. only, or say about 5 ft. per mile, and this can be distributed by making some lines 2½ ft. per mile too long, and others 2½ ft. per mile too short. The idea of an error of 250 ft. in a length of 100 miles is enough to break the heart of a computer, but when it is remembered that on a sheet of the 4 miles to an inch Ordnance Survey, which takes in 90 miles, it represents a length of about $\frac{1}{80}$ of

an inch, it will be seen that for map-making purposes it is inappreciable. In calculating the positions of the main points of a survey, no time or trouble should be spared to obtain correct results, and the positions stated in latitude and longitude are generally worked out to 3 decimals of a second of arc. These results can then be taken as starting points for new calculations, and so a survey of enormous size can be carried out without any accumulations of errors; but when it comes to laying down results on paper, seeing that errors do not accumulate, and recognising clearly the fact that there must be distortion, it is foolish to worry about errors which are of no practical importance.

Having arrived, then, at the conclusion that a map may be made without undue distortion embracing 5° of latitude, it may be interesting to consider the case of such a map extending, say, from 50° to 56° latitude which will practically include England, Wales, and, Ireland. This is a little over the 5° fixed upon, but the difficulty can be got over by arranging the projection as if it were between 50½° and 55½°, leaving the half degree at each end to look after itself, seeing that only very small areas of Cornwall and Northumberland will be affected.

The number of degrees of longitude taken in is immaterial, and the sheet will be as in fig. 56.

In this case the spheroidal form of the earth must be taken into account. Let each degree of the parallels of 51½° and 54½° be made the true length—viz., 227,800 ft. and 212,535 ft. The convergence amounts to 15,265 ft. The distance between them amounts to the sum of degrees 52,

53, 54 on the meridian, or say 1,095,230 ft., and the meridians will therefore meet at a distance of 15,248,900 ft.

FIG. 56

from parallel $54\frac{1}{2}°$, and the point of meeting will be the centre of the circles representing the parallels.

The lengths of the various degrees on the meridian are as follows:

Lengths of degrees of Meridian

50° to 51°	364,920 ft.
51° to 51½°	182,484 ,,
51½° to 52°	182,500 ,,
52° to 53°	365,046 ,,
53° to 54°	365,108 ,,
54° to 54½°	182,576 ,,
54½° to 55°	182,592 ,,
55° to 56°	365,228 ,,

To construct the map take any line A B (fig. 56) to represent the meridian of 4° west, and from A set off the distances given in the above table to mark the crossings of the various parallels, using the scale to which the map is to be plotted. Then with the same scale mark off along A B produced a distance of 15,249,060 ft. from the crossing of the parallel of 54½°. This will be the centre of the circles representing parallels.

Describe the parallels, showing 51½° and 54½° in pencil lines and the others in ink. Then mark off the true length of degrees along parallel 51½° as given in the table, extending the map to 11° west and 2° east, and draw meridians through the points of division to the centre of the circle. The lengths of degrees at the various latitudes will then work out as follows:

Lengths of degrees of Longitude

Latitude	on Map	on Earth	Difference per cent.
50°	235,432	235,198	0·10
51°	230,344	230,283	0·03
51½°	227,800	227,800	0
52°	225,256	225,298	0·02
53°	220,168	220,243	0·03
54°	215,080	215,121	0·02
54½°	212,535	212,535	0
55°	209,991	209,932	0·03
56°	204,903	204,679	0·11

From this table it will be seen that with the exception of parallels 50 and 56, which are really outside the map, none of the lengths in longitude are in error by an amount greater than 0·03 per cent., or about $\frac{1}{80}$ of an inch on a sheet 40 ins. long, which is inappreciable. Lengths north and south are correct.

A large-scale sheet plotted on its own projection, embracing an area between the parallels of 52° and 53° and 2° of longitude, will measure by its own scale top line 440,486 ft., bottom line 450,596 ft., and length north to south 365,046 ft.

If this sheet be reduced to a scale of $\frac{1}{1000000}$ it will measure as in the following table, while the graticule into which it has to fit will measure slightly less:

	Reduced sheet	Graticule
Top line	5·2858 in.	5·2840 in.
Bottom line	5·4071 ,,	5·4061 ,,
Length N. and S. ...	4·3805 ,,	4·3805 ,,

From this it will be seen that the difference between length of reduced sheet and length of graticule in the middle of the large map is only about 0·033 per cent., which is quite inappreciable, and something even less at latitudes 51° and 55°, and at the extreme edge beyond the limits of the actual map it is only about 0·11 per cent., an almost imperceptible amount.

It then follows that if large-scale maps are made, each sheet can be plotted on its own central meridian, with curved top and bottom lines and straight end lines. The various main points can be plotted on it from the computer's calculations with the utmost exactness, and each sheet, no matter what scale, can be reduced by

photography so as to fit into its place on the smaller scale. Reduction from one projection to another is of two sorts. In certain cases of reduction, such as those dealt with in the earlier chapters, the distortion caused by the projection on a plane surface necessitates altering the shape as well as the size of the large-scale map. When this is the case, lines of latitude and longitude must be ruled very close together on the large-scale map and on the projection, and the outlines must be filled in by a draughtsman by eye. In the case now being considered, however, there is no change of shape, or such a small change that it may be disregarded, and photography may be used. In the case of photography two varieties of reduction occur. There, is, first, the case where the small-scale map contains all the details of the large scale, as in the case of the 25-in. maps and 6-in. maps of the British Ordnance Survey. Here it is only necessary to take care that the lines on the large-scale map are heavy, and the lettering bold.

In most cases, however, the amount of detail to be shown on the small scale is less than that on the large scale, and then the system followed is to take an impression of the large-scale map in light blue colour. The roads or other details which are to be shown on the small scale are then drawn in, in heavy lines with black ink, and large bold lettering added. When this sheet is photographed the light blue does not come out, but only the black, and the general outline of the survey is thus well preserved without the detail. This system, it will be seen, is only applicable when the shape of the small-scale sheet is of the same shape as the large-scale sheet,

either exactly or within such narrow limits that the difference may be considered negligible, and the figures given above show that in the case of a map of England and Wales and Ireland, plotted on the conical projection above described, this relation of identity of shape within the specified limits will hold good if each of the large-scale sheets is plotted on conical projection on its own meridian.

There is this further point to be noted. While it is true that at the extreme north and south of the map the difficulty of fitting in sheets begins to show itself, the north sheet will nevertheless fit quite well into a map extending, say, from 53° to 58°, and so the following general statement is arrived at.

If in the survey of a country based on triangulation the latitude and longitude of each corner of a triangle be fixed with the greatest possible accuracy ; if the sheets on a large scale be plotted, each sheet bounded by meridians and parallels, and each be plotted on its own central meridian—then sheets can be reduced by photography or similar means so as to make up a sheet on a smaller scale embracing 5° of latitude, and this applies to any part of the area ; that is to say, one set of sheets can be used to take in latitude 40° to 45°, and another to make up a map taking in latitude 45° to 50°, and half of the same sheets from each set will make up a map taking in $42\frac{1}{2}°$ to $47\frac{1}{2}°$.

The maps referred to in this chapter are all supposed to be founded on triangulation, and some of the issues are sure to be on a large scale. If the system here treated of is adopted, each sheet is dealt with sepa-

rately, and there are then at hand materials for any smaller scale, and by reduction they can be made into maps containing 5° of latitude. On a scale of $\frac{1}{1000000}$ this is about the largest map that can be plotted on one sheet of paper, and such maps taking in about 8°, 10°, or 12° of longitude may form sheets of a world map.

When a map of a larger area than 5° of latitude has to be dealt with the projection has to be settled on the basis of the principles dealt with in the preceding chapter, the lines of latitude and longitude have to be laid down and the detail taken from the large-scale sheet by draughtsmen, but the particular projection to be adopted need not affect, *and ought not to affect*, the plotting of any sheet on a scale of $\frac{1}{1000000}$ or any larger scale.

In this respect the system differs essentially from the system of rectangular co-ordinates where the question of projection affects all scales *ab initio*.

If the account of the methods of dealing with small areas appears to be somewhat confused and unsatisfactory, it must be remembered that it is a subject which it is almost impossible to treat in a superficial manner, depending as it does to such a great extent on accuracy of detail.

Indeed, if anyone takes up a complete work on the subject with a view to getting general ideas, he will probably give up the attempt before he has mastered the rules for distributing slight fractional errors of observation in the triangulation among the various angles affected, but it is hoped that the following statements will be accepted.

(1) That it is possible by choosing a convenient piece of flat ground, and taking every possible precaution, to measure a base line of some miles in length within limits of error which are inappreciable (generally 2 or 3 in.).

(2) That it is possible by a system of triangulation to extend this measurement so as to get the length of an arc of the meridian with practical accuracy.

(3) That by noting and comparing measurements of arcs in various parts of the world, it is possible to determine the size and shape of the world, and that this has now been done with tolerable accuracy, and that the regular tables of lengths of degrees, &c., re-calculated on this basis, are sufficiently in accordance with facts to justify their acceptance at present as the basis of further calculations.

(4) That it is possible on the above basis to extend a system of triangles over a very large area with so much accuracy that it is reasonable to state the latitude and longitude of the various corners of the triangles to 3 places of decimals of a second of arc. (In the case of very large surveys it is, of course, the practice to check the result by measuring one or more of the calculated lines.)

(5) That within certain limits it is possible to plot the positions of these points, and fill in the details of the survey of a piece of country with no appreciable error.

(6) That an area of 1° of latitude with a length of, say, 100 miles east and west, is well within these limits, such an area making a convenient sheet on a scale of 4 miles to an inch, on which scale roads and all important details can be shown.

(7) That an area of 5° of latitude with, perhaps, 500 miles length east and west, or, say, a convenient sheet on a scale of $\frac{1}{1000000}$, or about 16 miles to an inch, can be plotted with sufficient accuracy considering the very small amount of detail that can be shown, but that in all larger areas the necessary and ineradicable errors of projection must show themselves.

In the author's opinion it follows that, seeing that what may be considered absolutely correct maps can be made of considerable areas, no other maps on a large scale should ever be made, and that this involves plotting each sheet of the large scale on its own projection without reference to adjoining sheets, and preferably bounding each by meridians and parallels. When a map of a large area on a small scale is required, some of the projections referred to in the earlier chapters must be adopted, the detail filled in as there described, and if the map is too large to be printed on one sheet, it must be cut up into convenient sized sheets, but the projection will be that of the entire map, and will have no reference to the sheets into which it is divided.

It is hoped that the preceding pages will serve the purpose intended—viz., will enable anyone without mathematical knowledge to understand various projections in a general way, and recognise the advantages and defects of each, and likewise will enable anyone with a fair knowledge of elementary mathematics to understand the mathematical basis of the various projections referred to.

It is likewise hoped that the treatise may be useful both as a text-book to young students who do not require

to go deeply into the subject, and as an elementary text-book to those whose profession requires them to master map projection in all its details, and to whom the more advanced text-books might at first be rather puzzling.

The table on the following page gives the lengths of a degree of latitude and a degree of longitude at each degree of latitude on the assumption that the earth is an ellipsoid, of which the equatorial semi-diameter is 20,923,600 ft. and the polar semi-diameter is 20,853,657 ft.

Geograph. Latitude	Degree of Meridian	Degree of Parallel	Geograph. Latitude	Degree of Meridian	Degree of Parallel
°			°		
0	362748·33	365185·71	45	364571·77	258657·25
1	749·43	5130·47	46	635·68	4118·51
2	752·75	4964·74	47	699·54	249501·81
3	758·28	4688·57	48	763·25	4808·54
4	766·00	4302·05	49	826·75	0040·09
5	775·91	3805·29	50	889·96	235197·90
6	788·01	3198·43	51	952·80	0283·42
7	802·27	2481·64	52	365015·20	225298·13
8	818·68	1655·13	53	077·06	0243·54
9	837·22	0719·13	54	138·34	215121·16
10	857·86	359673·92	55	198·93	209932·55
11	880·59	8519·79	56	258·78	4679·26
12	905·37	7257·07	57	317·80	199362·90
13	932·18	5886·13	58	375·93	3985·08
14	960·98	4407·36	59	433·10	188547·43
15	991·74	2821·19	60	489·23	3051·59
16	363024·42	1128·07	61	544·25	177499·25
17	058·99	349328·50	62	598·10	1892·10
18	095·40	7422·98	63	650·70	166231·84
19	133·61	5412·07	64	702·00	0520·21
20	173·57	3296·36	65	751·94	154758·95
21	215·23	1076·45	66	800·44	148949·83
22	258·56	338752·98	67	847·45	3094·63
23	303·49	6326·62	68	892·91	137195·14
24	349·96	3798·08	69	936·77	1253·18
25	397·93	1168·10	70	978·97	125270·57
26	447·34	328437·44	71	366019·45	119249·14
27	498·13	5606·89	72	058·17	3190·78
28	550·24	2677·27	73	095·08	107097·32
29	603·60	319649·44	74	130·14	100970·66
30	658·14	6524·29	75	163·30	94812·70
31	713·81	3302·72	76	194·51	88625·32
32	770·54	309985·68	77	223·74	82410·44
33	828·27	6574·14	78	250·96	76169·97
34	886·91	3069·10	79	276·13	69905·87
35	946·40	299471·60	80	299·21	63620·07
36	364006·67	5782·67	81	320·19	57314·53
37	067·64	2003·42	82	339·02	50991·17
38	129·24	288134·97	83	355·69	44651·98
39	191·40	4178·44	84	370·19	38298·93
40	254·04	0135·01	85	382·49	31933·97
41	317·08	276005·89	86	392·57	25559·08
42	380·45	1792·28	87	400·43	19176·27
43	444·08	267495·45	88	406·04	12787·49
44	507·87	3116·67	89	409·42	6394·74
45	571·77	258657·25	90	410·54	0000·00

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